PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 60 JANUARY, 1934 NO. 1

TECHNICAL PAPERS
DISCUSSIONS
APPLICATIONS FOR ADMISSION AND TRANSFER

Published monthly, except June and July, at 99-129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

Copyright, 1934, *by the* AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Foundation Treatment at Rodriguez Dam. *Charles P. Williams*	Oct., 1932	
Discussion	Mar., 1933	Closed
Study of Stilling Basin Design. *C. Maxwell Stanley*	Nov., 1932	
Discussion	Apr., 1933	Closed
Wind Stress Analysis Simplified. *L. E. Grinter*	Jan., 1933	
Discussion (Author's closure)	Apr., May, Sept., 1933. Jan., 1934	Closed
Evaporation from Water Surfaces: A Symposium	Feb., 1933	
Discussion	May, Aug., Nov., Dec., 1933	Closed
The Plan of Boston, Massachusetts: A Capital City. *Arthur C. Comey*	Mar., 1933	
Discussion	May, 1933	Closed
High Dams on Pervious Glacial Drift. *Edward M. Burd*	Apr., 1933	
Discussion	May, Sept., Oct., Nov., 1933	Closed
Improved Type of Flow Meter for Hydraulic Turbines. *Ireal A. Winter*	Apr., 1933	
Discussion	Aug., Nov., 1933	Closed
Actual Deflections and Temperatures in a Trial-Load Arch Dam. *A. T. Larned and W. S. Merrill*	May, 1933	
Discussion	Sept., Oct., Nov., Dec., 1933	Feb., 1934
Wind Stresses by Slope Deflection and Converging Approximations. *John E. Goldberg*	May, 1933	
Discussion	Aug., 1933. Jan., 1934	Feb., 1934
Progress Report of Special Committee on Earths and Foundations	May, 1933	
Discussion	Aug., Sept., Oct., Nov., Dec., 1933, Jan., 1934	Uncertain
Water Power Development of the St. Lawrence River. *Daniel W. Mead*	Aug., 1933	
Discussion	Aug., Nov., Dec., 1933	Feb., 1934
On the Behavior of Siphons. *J. C. Stevens*	Aug., 1933	
Discussion	Dec., 1933	Feb., 1934
Use and Capacity of City Streets. *Hawley S. Simpson*	Aug., 1933	
Discussion	Nov., 1933	Feb., 1934
Stability of Straight Concrete Gravity Dams. *D. C. Henny*	Sept., 1933	
Discussion	Nov., Dec., 1933, Jan., 1934	Feb., 1934
Estimating the Economic Value of Proposed Highway Expenditures. *Thomas R. Agg*	Sept., 1933	
Discussion	Nov., Dec., 1933, Jan., 1934	Feb., 1934
The Surveyor and His Legal Equipment. *A. H. Holt*	Sept., 1933	
Discussion	Nov., Dec., 1933, Jan., 1934	Feb., 1934
Photo-Elastic Analysis of Stresses in Composite Materials. *A. H. Beyer and A. G. Solakian*	Sept., 1933	
Discussion	Jan., 1934	Feb., 1934
Water-Bearing Members of Articulated Buttress Dams. *Hakan D. Birke*	Sept., 1933	Feb., 1934
Duration Curves. *H. Alden Foster*	Oct., 1933	
Discussion	Dec., 1933, Jan., 1934	Mar., 1934
Analysis of Unsymmetical Concrete Arches. *Charles S. Whitney*	Oct., 1933	Mar., 1934
Deformation of Steel Reinforcement During and After Construction. *Sergius I. Sergev*	Oct., 1933	
Discussion	Nov., 1933, Jan., 1934	Mar., 1934
Intercepting Sewers and Storm Stand-By Tanks at Columbus, Ohio. *John H. Gregory, R. H. Simpson, Orris Bonney, and Robert A. Allton*	Oct., 1933	Mar., 1934
Some Soil Pressure Tests. *H. de B. Parsons*	Nov., 1933	
Discussion	Jan., 1934	Mar., 1934
Lincoln Highway from Jersey City to Elizabeth, N. J. *Sigvald Johannesson*	Nov., 1933	Mar., 1934
Practical River Laboratory Hydraulics. *Herbert D. Vogel*	Nov., 1933	Mar., 1934
Formation of Floc by Ferric Coagulants. *Edward Bartow, A. P. Black, and Walter E. Sansbury*	Dec., 1933	Mar., 1934
Modifying the Physiographical Balance by Conservation Measures *A. L. Sonderegger*	Dec., 1933	Mar., 1934
Model of Calderwood Arch Dam. *A. V. Karpov, and R. L. Templin*	Dec., 1933	Mar., 1934

CONTENTS FOR JANUARY, 1934

PAPERS

PAGE

An Approach to Determinate Stream Flow.
By Merrill M. Bernard, M. Am. Soc. C. E. 3

Discharge Formula and Tables for Sharp-Crested Suppressed Weirs.
By C. G. Cline, Esq. 19

Renewal of Miter-Gate Bearings, Panama Canal.
By Clinton Morse, Jun. Am. Soc. C. E. 37

Loss of Head in Activated Sludge Aeration Channels.
By Darwin Wadsworth Townsend, M. Am. Soc. C. E. 63

Williot Equations for Statically Indeterminate Structures in Combination with Moment Equations in Terms of Angular Displacements.
By Charles A. Ellis, M. Am. Soc. C. E. 73

DISCUSSIONS

Wind Stress Analysis Simplified.
By L. E. Grinter, Assoc. M. Am. Soc. C. E. 93

Wind Stresses by Slope Deflection and Converging Approximations.
By Messrs. S. S. Gorman, and A. Floris 103

Progress Report of Special Committee on Earths and Foundations.
By Frederic R. Harris, M. Am. Soc. C. E. 107

Stability of Straight Concrete Gravity Dams.
By Messrs. William P. Creager, F. W. Hanna, Lars R. Jorgensen, and I. M. Nelidov 112

Estimating the Economic Value of Proposed Highway Expenditures.
By J. T. L. McNew, M. Am. Soc. C. E. 125

CONTENTS FOR JANUARY, 1934 (*Continued*)

PAGE

The Surveyor and His Legal Equipment.
By Messrs. Clarence T. Johnston, A. F. Harley, and Lynn Perry 128

Photo-Elastic Analysis of Stresses in Composite Materials.
By Messrs. H. J. Gilkey, and Robert V. Baud 136

Duration Curves.
By Messrs. C. R. Pettis, and W. G. Hoyt 140

Deformation of Steel Reinforcement During and After Construction.
By Messrs. F. E. Richart, and Homer M. Hadley 146

Some Soil Pressure Tests.
By J. C. Meem, M. Am. Soc. C. E. 150

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

The Society is not responsible for any statement made or opinion expressed in its publications

MEMBERSHIP

Applications for Admission and Transfer ..... following p. 154

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

AN APPROACH TO DETERMINATE STREAM FLOW

By Merrill M. Bernard,[1] M. Am. Soc. C. E.

Synopsis

In a paper on the determination of stream flow from rainfall records, LeRoy K. Sherman, M. Am. Soc. C. E.[2] undertook, through the summation of daily increments of run-off, not only to establish the maximum flow resulting from a recorded rainfall, but to produce the stream hydrograph, a process that to a remarkable degree parallels the phenomenon of stream flow produced from storm run-off.

The results of a study, the outgrowth of an interest in Mr. Sherman's paper, is presented herewith. It treats of other aspects of the problem and offers a procedure that is believed to be an approach to determinate stream flow. The transition from rainfall to stream flow is accomplished through the medium of a "distribution graph," which is found to be a function of water-shed characteristics. The possibility of developing the distribution graph without resorting to the stream-flow records is shown. The "pluviagraph," or maximum hydrograph, is discussed, and its value to design in the field of hydraulic engineering is demonstrated.

The study utilizes six water-sheds of from 500 to 6 000 sq miles and embraces practically the entire State of Ohio. These water-sheds were selected from the unregulated streams served by the United States Geological Survey, and represent an acceptable range in area and water-sheds characteristics.

Only such illustrations are included as are necessary to demonstrate the points raised in the paper. A complete copy of the original paper, with all data and drawings, is filed for reference in Engineering Societies Library, 29 West 39th Street, New York, N. Y.

Note.—Discussion on this paper will be closed in April, 1934, *Proceedings*.

[1] Cons. Civ. Engr., Crowley, La.

[2] "Streamflow from Rainfall by Unit-Graph Method," *Engineering News-Record*, April 7, 1932.

Introduction

A general interest in the flow of a stream divides itself into: (1) The conservation of water for such useful purposes as irrigation and power development; and (2) the disposal of flood-waters without loss or damage. In the latter case the objective will be the determination of maximum values of flood volume or stream stage, depending on the nature of the problem and the degree of protection sought.

The ideal situation is that of finding a long-term flow record applicable to the location of the structure to be designed. If the project is one of flood protection, however, or of channel improvement, that disturbs the regimen of the stream; or if the water-shed is subject to further development, even this record must be modified to meet the new conditions of delivery and concentration of flood-water.

As is more generally the case, the designer is forced to deal with a short-term record on the stream itself, must extrapolate a record of a neighboring stream having similar water-shed characteristics, or must attempt to establish the magnitude and frequency of various flood flows by analyzing the factors that produce them. In dealing with such problems, complexities have discouraged all except a comparatively superficial treatment of the problem.

The evolution of this science is interesting. The best of the older empirical formulas expressed flood flow as a function of area only; others introduced rainfall rate and slope factors. More recent studies are to be divided into two groups. One group, utilizing recorded stream flow, relates flood flow to area by means of coefficients and exponents which tend to become constant for localities of considerable extent. The other group attempts to express flood flow as a function of storm intensity, more recently giving values to such factors as storm frequency and duration, and water-shed characteristics. Further development of the former method is restricted by a data source rendered inadequate through poor distribution and short-record period. The latter group, advocating the rational method, or some modification of it, must develop, through research, values for several important factors now indeterminate.

The objective in this field may be stated thus; A method of determining flood flow in terms of magnitude, duration, and frequency, at any point on any stream, within a region having the same range in climatological, topographical, hydrological, and geological conditions. The method must give determinate values to all principal factors affecting run-off and must group the minor influences into coefficients that approach constant values of regional application. It must utilize available data only, which, in this country, are limited to the published records of the United States Weather Bureau and the United States Geological Survey. The method must meet the test of reproducing the stream hydrograph accurately.

Basic Data

The foregoing "specification" has stipulated that the proposed method must be capable of utilizing published Weather Bureau and Geological Sur-

vey records. This assumes their application to the larger water-sheds on which the time unit for the unit graph or for the distribution graph is one day or more. The advantages of general availability and distribution far outweigh the disadvantages and objections which are set forth, with suggested remedies, in a recent Committee report.[3] Other improvements in this service, admittedly not easy of accomplishment, would be: (1) All stations to take readings on the same hour; (2) notation of hour and depth at end of period of continuous downpour, within the 24-hr interval; and (3) improved typography and arrangement in the monthly publication of climatological data.

The Unit Graph.—Defining the theory of the unit graph, LeRoy K. Sherman, M. Am. Soc. C. E., states[4] that, for the same drainage area there is a definite total flood period corresponding to a given rainfall, and that the length of the base of the hydrograph will be the same for all 1-day rainfalls, regardless of intensity.

According to Mr. Sherman, "if a given one-day rainfall produces a 1-in. depth of run-off over the given drainage area, the hydrograph showing the rates at which the run-off occurred can be considered a unit-graph for that water-shed."

The unit graph is constructed by segregating the daily stream flow resulting from a single downpour of 1 day (or the other time unit), from the flow occasioned by previous and subsequent rainfall. This, by proportion, is converted into run-off, in feet per second, equivalent to a depth of 1 in. over the area, and becomes the unit graph of the water-shed.

The hydrograph is developed through the following procedure: (*a*) Daily rainfall, properly weighted, is listed by date; (*b*) a coefficient is applied, reducing rainfall to run-off, in inches of depth; (*c*) the unit-graph value for each day, multiplied by the run-off depth, gives the increment of stream flow attributable on that day to the daily rainfall so affected; and (*d*) the horizontal summation of the run-off increments, day by day, produces the total flow and becomes the ordinates of the hydrograph.

The Distribution Graph.—The writer's conception of the theory on which the unit graph is based, is as follows: For a particular water-shed, two storm periods confined to the same time interval and uninfluenced by preceding and subsequent rainfall will become effective to stream flow at the same proportional rate, expressed as a percentage of the area contributing its run-off to the flow of the stream, at the point of observation, day by day. This is to be conceived as a time contour, which, having developed around the point of concentration, recedes to the outer limits of the water-shed, progressively enveloping the various portions as they contribute to the flow, reaching the extremities when all parts have contributed and the area under the distribution graph is equal to 100 per cent.

Here, as in the unit-graph method, the concentration period may be defined as a time interval, at the end of which all parts of the water-shed

[3] *Proceedings*, Am. Soc. C. E., January, 1933, p. 153.

[4] "Streamflow from Rainfall by Unit-Graph Method," *Engineering News-Record*, April 7, 1932, p. 50

are contributing to the flow at the point of observation. This definition should apply, as well, to the time interval used in the rational method to establish the critical average rainfall intensity; but it will be found that the concentration period, as usually estimated or calculated, is much shorter. The calculated time in transit of flood-waters from head-water to outlet on the Tuscarawas River, at Newcomerstown, Ohio, is between 3 000 and 4 000 min, while its distribution graph shows that only about one-half its area has become effective to the stream flow in that period. The error lies in a misconception of the meaning of "remote area." It is usual to consider that part of the water-shed in the vicinity of the head-waters as establishing concentration time because of its distance from the outlet. As a matter of fact, the efficiency of the main channel, through higher velocities, places head-water areas much closer to the outlet in terms of time than extensive areas nearer but less advantageously located.

It is difficult to select from the rainfall record, a storm of the type which will produce a hydrograph suited to the construction of the distribution graph of the water-shed. The task requires boundless patience and the application of unusual care and judgment. Upon first attempt one is convinced that any result can be obtained. A storm period, to produce the distribution graph of the water-shed, must have the following characteristics: (1) The rain must have fallen within the recording time interval, or the time unit, as the day (or hour); (2) the storm must have been well distributed over the water-shed, all stations showing an appreciable depth of rainfall; (3) the run-off following the storm must have been uninterrupted by the effects of low temperatures and unaccompanied by melting snow and ice; and (4) the storm period must occupy a place of comparative isolation in the record. It should follow a period of low stream flow and there should be no further rainfall until the flood peak is well passed.

Certain unavoidable sources of error and inaccuracy must be recognized and accepted in dealing with Weather Bureau data. At some rainfall stations readings are made at 8:00 A. M.; at others, at 5:00 P. M. It is highly probable that at many of the co-operative stations readings are taken at irregular intervals, the record going in as a forenoon or an afternoon observation. A continuous storm of 24 hr, or less, may be divided by the recording hour, appearing in the record as a 2-day storm. Particularly on small water-sheds it is possible that the entire rainfall may have fallen in the first hour, the flood peak on the twelfth hour, and the flow, officially recorded on the twenty-fourth hour, may be only a small fraction of the maximum. Stream flow is recorded as a 24-hr average, obscuring true maximum values, particularly on small water-sheds.

Table 1 includes all data used in developing the distribution graphs for the six water-sheds embraced by this paper. In every case but one at least three storm periods were found which gave acceptable graphs, the average being taken as the distribution graph for the water-shed.

TABLE 1.—Arrangement of Data for Development of Distribution Graphs

(Units in Cubic Feet per Second)

Storm No. 1					Storm No. 2					Storm No. 3					Distribution Graph, in Percentage of Total Flow on the Water-Shed	
(a) Raccoon Creek, at Adamsville, Ohio																
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
April, 1925:					June, 1927:											
21	282	282	0	.....	18	530	530	0	.....	.....	.....	.....	.....	.....	.....	.....
22	2 150	250	1 900	17	19	1 300	400	900	15	.....	.....	.....	.....	.....	16	17
23	3 290	225	3 065	28	20	2 070	330	740	29	.....	.....	.....	.....	.....	29	32
24	2 840	198	2 642	24	21	1 720	290	1 430	24	.....	.....	.....	.....	.....	24	16
25	1 300	170	1 130	10	22	950	252	698	12	.....	.....	.....	.....	.....	11	9
26	750	125	625	6	23*	540	229	311	5	.....	.....	.....	.....	.....	5	6
27*	550	75	475	4	24	405	200	205	3	.....	.....	.....	.....	.....	4	4
28	410	40	370	3	25	335	172	163	3	.....	.....	.....	.....	.....	3	3
29	340	36	304	3	26	290	140	150	3	.....	.....	.....	.....	.....	3	3
30	290	29	261	2	27	255	123	132	2	.....	.....	.....	.....	.....	2	2
May, 1925:																
1	255	24	231	2	28	230	107	123	2	.....	.....	.....	.....	.....	2	2
2	219	19	200	1	29	200	86	114	2	.....	.....	.....	.....	.....	1	1
.....	.....	.....	11 203	100	.....	.....	.....	4 966	100	.....	.....	.....	.....	.....	100	95
(b) Licking River, at Toboso, Ohio																
July, 1924:					July, 1923:					June, 1924:						
12	218	218	0	.....	2	130	130	0	.....	22	374	374	0	.....	.....	.....
13	2 600	200	2 400	65	3	1 680	122	1 558	66	23	3 400	277	3 123	68	66	66
14	630	190	440	12	4	457	117	340	14	24	790	243	557	13	13	16
15	600	180	320	9	5	261	114	147	7	25	541	226	315	7	8	8
16	374	170	204	6	6	198	107	92	4	26	401	203	198	4	5	5
17	277	164	113	3	7	176	104	72	3	27*	374	190	184	3	3	3
18	243	160	83	2	8*	162	104	58	2	28	277	180	97	2	2	2
19	226	153	73	2	9	156	104	52	2	29	243	170	73	2	2	2
20	203	146	57	1	10	130	100	30	2	30	226	160	66	1	1	1
.....	.....	.....	3 690	100	.....	.....	.....	2 349	100	.....	.....	.....	4 613	100	100	103

* Day record abandoned.

TABLE 1.—(*Continued*)

Storm No. 1					Storm No. 2					Storm No. 3					Distribution Graph, in Percentage of Total Flow on the Water-Shed	
(c) Hocking River, at Athens, Ohio																
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
June, 1927:					June, 1927:					Oct., 1926:						
3	435	435	0	.....	17	1 380	1 380	0	.....	1	740	740	0	.....	.....	.....
4	1 100	400	700	11	18	2 360	900	1 460	10	2	1 240	650	590	8	10	14
5	2 780	375	2 405	37	19	7 100	700	6 400	45	3	4 120	520	3 600	44	42	34
6	1 940	350	1 590	26	20	3 560	530	3 030	22	4	2 290	470	1 820	23	24	17
7	770	320	450	8	21	1 660	470	1 190	8	5	1 100	420	680	8	8	10
8*	535	310	225	4	22	1 170	420	750	5	6*	895	380	515	6	5	6
9	480	290	190	3	23*	900	390	510	4	7	600	340	260	3	3	4
10	425	242	183	3	24	700	360	340	2	8	510	310	200	2	2	3
11	397	220	177	3	25	530	340	190	2	9	460	290	170	2	2	3
12	370	200	170	3	26	470	320	150	1	10	410	260	150	2	2	2
13	345	181	164	2	27	420	310	110	1	11	380	240	140	2	2	2
.....	.....	.....	6 254	100	.....	.....	.....	14 130	100	.....	.....	.....	8 125	100	100	95
(d) Walhonding River, at Pomerene, Ohio																
July, 1922:					July, 1927:					June, 1927:						
18	582	582	0	.....	21	502	502	0	.....	3	1 660	1 660	0	.....	.....	.....
19	633	540	93	9	22	1 010	465	545	11	4	3 470	1 320	2 150	11	11	9
20	905	475	430	42	23	2 280	438	1 842	38	5	11 100	1 210	9 890	48	41	41
21	599	440	159	16	24	1 160	408	752	16	6	3 830	1 060	2 770	14	16	21
22	499	410	89	9	25	775	360	415	9	7	2 410	890	1 520	7	8	12
23*	460	385	75	7	26	608	330	278	6	8	1 900	810	1 090	5	6	8
24	397	340	57	5	27*	536	310	226	5	9	1 540	765	775	4	5	5
25	360	320	40	4	28	470	300	170	4	10	1 320	720	600	3	4	4
26	330	300	30	3	29	438	280	158	3	11	1 210	635	575	3	3	3
27	310	290	20	2	30	408	250	148	3	12	1 060	605	455	2	3	2
28	300	280	20	2	31	360	220	140	3	13*	890	580	310	2	2	2
29	290	275	15	1	Aug., 1927:				2	14	810	530	280	1	1	1
.....	.....	.....	.....	.....	1	330	195	135	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	1 028	100	.....	.....	.....	4 809	100	.....	.....	.....	20 415	100	100	108

* Day record abandoned.

TABLE 1.—(Continued)

Storm No. 1					Storm No. 2					Storm No. 3					Distribution Graph, in Percentage of Total Flow on the Water-Shed	
(e) Tuscarawas River, at Newcomerstown, Ohio																
Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Date	Observed stream flow	Base flow	Segregated flow	Distribution (percentage of total segregated flow)	Average of Storms Nos. 1, 2, and 3	Taken from Fig. 2
July, 1922:					May, 1925:					June, 1927:						
17	542	542	0	.....	23	980	980	0	.....	4	1 560	1 560	0	.....	.....	.....
18	644	485	159	4	24	1 080	745	335	4	5	3 110	1 420	1 690	7	5	6
19	1 540	420	1 120	25	25	2 280	615	1 665	19	6	8 250	1 315	6 935	26	24	24
20	1 300	360	940	21	26	2 280	505	1 775	20	7	7 490	1 245	6 245	24	23	25
21	886	330	556	13	27	1 530	419	1 111	13	8	4 530	1 200	3 330	13	13	14
22	682	315	367	8	28	1 240	367	873	10	9	3 280	1 100	2 180	8	8	9
23	609	298	311	7	29	1 030	355	675	8	10	2 440	1 035	1 405	5	6	6
24	542	280	262	6	30	932	319	613	7	11	2 120	985	1 135	4	5	5
25*	485	270	215	5	31	790	308	482	5	12	1 830	935	895	3	4	4
26	420	265	155	4	June, 1925:				4	13*	1 590	900	690	3	4	3
27	360	260	100	2	1	700	300	400	4	14	1 440	870	570	2	3	2
28	330	255	75	2	2*	600	290	310	3	15	1 330	855	475	2	2	2
29	315	250	65	2	3	550	280	270	.....	16	1 250	830	420	2	2	2
30	298	250	48	1	4	450	270	180	2	17	1 210	800	410	1	1	1
.....	.....	.....	.....	.....	5	400	260	140	1	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	4 373	100	.....	.....	.....	8 829	100	.....	.....	.....	26 380	100	100	103
(f) Muskingum River, at Dresden, Ohio																
July, 1927:					July, 1925:					Feb., 1929						
21	1 680	1 680	0	.....	9	2 390	2 390	0	.....	25	2 520	2 520	0	.....	.....	.....
22	2 140	1 500	640	2	10	3 560	2 200	1 360	3	26	18 700	2 300	16 400	5	3	3
23	4 950	1 370	3 580	13	11	7 990	2 100	5 890	15	27	45 600	2 150	43 450	14	14	11
24	7 450	1 270	6 180	22	12	10 600	1 960	8 640	22	28	52 200	2 030	50 170	16	20	24
25	6 070	1 160	4 910	18	13	8 920	1 850	7 070	18	Mar., 1929						
26	3 990	1 050	2 940	11	14	6 400	1 720	4 680	12	1	50 300	1 910	48 390	15	11	19
27	2 660	980	1 680	6	15	3 980	1 500	2 480	7	2	45 200	1 800	43 400	14	12	13
28	2 140	930	1 210	4	16	2 780	1 400	1 380	4	3	32 200	1 600	30 600	9	7	9
29*	2 000	870	1 130	4	17*	2 430	1 280	1 150	3	4	19 800	1 450	18 350	6	5	7
30	1 880	820	1 060	4	18	2 230	1 190	1 040	3	5*	17 000	1 350	15 650	5	4	5
31	1 800	780	1 020	4	19	2 110	1 080	1 030	3	6	14 500	1 230	13 270	4	4	4
Aug., 1927:					20	1 980	990	990	2	7	11 000	1 130	9 870	3	3	3
1	1 600	740	860	3	21	1 870	940	930	2	8	8 350	1 030	7 320	2	3	3
2	1 450	700	750	3	22	1 770	890	860	2	9	7 450	960	6 490	2	2	2
3	1 350	680	670	2	23	1 570	850	720	2	10	5 500	910	4 590	2	2	2
4	1 230	680	550	2	24	1 430	810	620	1	11	4 300	860	3 440	1	2	2
5	1 130	680	450	1	25	1 330	780	550	1	12	3 970	810	3 160	1	1	2
6	1 020	680	340	1	.....	.....	.....	.....	1	13	3 700	775	2 925	1	1	1
.....	.....	.....	27 970	100	.....	.....	.....	39 390	100	.....	.....	.....	317 475	100	100	110

*Day record abandoned.

The daily value of the distribution graph is given a proportional value instead of a quantitative value as in the unit graph. In developing it, the rainfall record is only necessary to check the isolation, duration, and distribution of the storm period and to determine the date on which subsequent rainfall affected the flow of the stream, at which point the record is to be abandoned and the further recession of the flood flow estimated.

By taking a number of storm periods followed by intervals of no rainfall, a composite "recession curve" can be prepared which gives a dependable idea of how the base flow would have fallen and the flood flow would have receded had no subsequent rainfall occurred.

The Water-Shed Factor

As the area under all distribution graphs is equal to 100%, the differences between water-sheds are reflected in the respective shapes of their distribution graphs. It is apparent, too, that the distribution graph demonstrates the inadequacy of area as the basis of comparison between water-sheds, as illustrated by Fig. 1. Here, arises the need for some means of evaluating

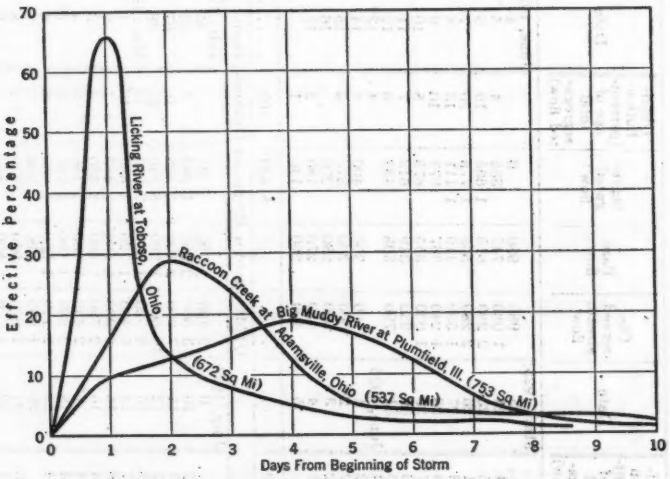

Fig. 1.—Distribution Graphs of Three Water-Sheds.

the characteristics of a water-shed which so markedly change the shape of the distribution graph on water-sheds of approximately the same area.

Elsewhere[5] the writer has segregated from the general expression of run-off[6] the factors representing water-shed characteristics into the expression:

$$U = \left(\frac{60\,P}{L}\right)^{4eq} F^{8eq}\,\frac{S^{1.5eq}}{1\,000^{2eq}} \qquad (1)$$

in which, P is a constant, depending on the shape of the area of the water-shed and its manner of concentration; L, the length, in feet, which water

[5] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1154.
[6] *Loc. cit.*, p. 1059.

must traverse in running from the most remote portion of the water-shed to its outlet; F, a constant, depending on the shape and condition of the main channel of flow; S, the fall, in feet per 1 000 ft of main channel of flow; and e, a positive fractional exponent of t, the duration of rainfall intensity for the locality.

Table 2 gives the values of the various factors entering into the determination of U in Equation (1). In its further use on large water-sheds the

TABLE 2.—COMPUTATION DATA

Description	Raccoon Creek	Licking River	Hocking River	Walhonding River	Tuscarawas River	Muskingum River
M, in square miles	537	672	944	1 490	2 430	5 980
L, in feet	380 000	234 300	401 200	475 000	486 000	706 000
P (a constant)	0.543	0.469	0.513	0.510	0.480	0.470
F (a constant)	4.85	4.79	4.69	4.56	4.41	4.30
S, in feet per 1 000 ft	0.89	2.22	1.05	0.64	0.65	0.62
V (a constant)	0.0600610	0.0001570	0.0000540	0.000042	0.0000320	0.0000195

factor, P, should be re-tabulated and its range of values checked. The values given for the factor, F, in Table 2 are known to be inaccurate, the writer having no direct data at hand regarding channel shape and condition. The values of L and S were taken from the topographic maps of the U. S. Geological Survey.

If the influences of the water-shed upon run-off can be separated from the influences of the storm, and if these water-shed influences are given a determinate value such as U, then it is possible to prepare a chart, as shown in Fig. 2, applicable to any region over which geological and climatological conditions are, in general, the same. This enables the designer, after having determined the value of U, to select, at any point on any stream within the region, a distribution graph for its water-shed. The value of this procedure to stream-flow problems, at points where no flow records are available, can be appreciated.

The relationships between the value of U, the daily value of the distribution graph, and the progressive duration of the storm period, for the region embraced by the study, can be stated as follows: For any particular day within the storm period the daily value of the distribution graph increases directly as the five-thirds power of U to a maximum daily value; then it recedes inversely as the two-thirds power of U. Furthermore, for a particular value of U, the daily value of the distribution graph increases directly as the square of the storm duration, in days, to a maximum value, then it recedes inversely as the square of the storm duration, in days.

Consistent results from the use of the chart are shown in Table 1, the last column in the table giving a distribution graph for each water-shed, the values being taken from the curves of the chart.

REPRODUCING THE HYDROGRAPH

The reproduced hydrograph serves the principal purpose of demonstrating the utility of the method proposed and is the means of developing coefficients.

Table 3 demonstrates the arrangement of the computation sheet for the Tuscarawas River, at Newcomerstown. The procedure is, as follows:

1.—Daily rainfall is taken from the monthly climatological record, weighted[7] according to station-area distribution and listed opposite the proper date in Column (2).

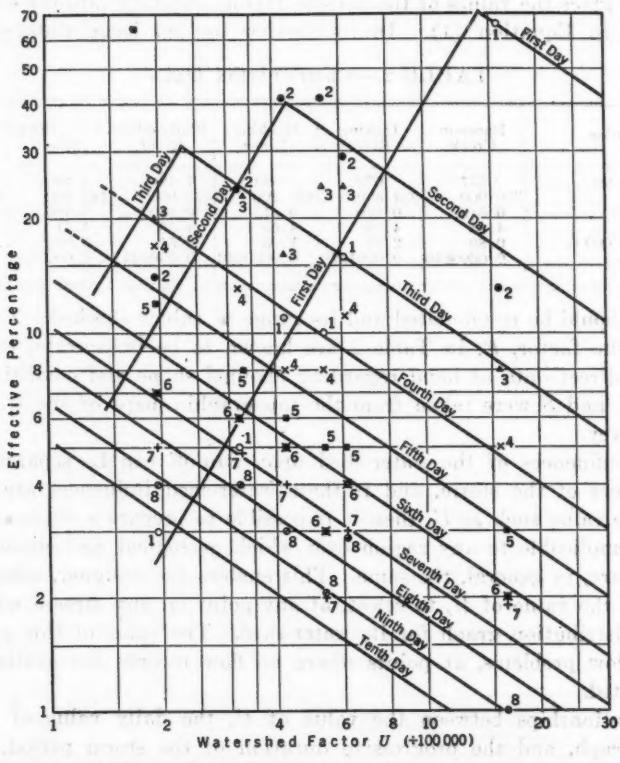

FIG. 2.—RELATION OF WATER-SHED FACTOR *U*, TO DAILY INCREMENTS OF THE VARIOUS DISTRIBUTION GRAPHS.

2.—The distribution graph is applied to the weighted daily rainfall and listed in Columns (3) to (13), inclusive, beginning the distribution opposite the date on which the rainfall occurred.

3.—The rainfall increments are added horizontally and the summation, the effective rainfall depth, *D*, is listed in Column (14).

4.—The 24-hr average observed stream flow is taken from the Geological Survey record and listed according to date in Column (15).

5.—Stream flow, in second-feet-days, is converted into depth of run-off, in inches, by the conversion equation,

$$D = \frac{0.03718\,Q}{M} \qquad (2)$$

[7] *Engineering News-Record*, August 2, 1917, p. 211.

TABLE 3.—COMPUTATION SHEET, TUSCARAWAS RIVER, AT NEWCOMERSTOWN, OHIO

Date	Weighted daily rainfall, in inches	Distribution Graph of the Water-Shed Applied to Weighted Daily Rainfall, as Follows: Day..... 1 2 3 4 5 6 7 8 9 10 11 12												Effective rainfall depth, I, in inches	24-hour average observed stream flow, Q, in feet per second	Run-off depth, Q = 0.03718 Q / square miles = 0.03718 Q / 2 430, in inches	Retention coefficient, $C = \frac{\text{Ave. } D}{\text{Ave. } I}$	Computed $D = CI$, in inches	Computed Q, in feet per second	Pluvigraph values, in feet per second
		Effective percentage.... 6 24 23 12 8 6 5 4 4 3 3 2																		
(1	(2	(3	(4	(5	(6	(7	(8	(9	(10	(11	(12	(13		(14	(15	(16	(17	(18	(19	(20
July 27.....	0.18	0.011																		
July 28.....	0.54	0.043	0.082																	
July 29.....	....	0.041	0.130																	
July 30.....	....	0.022	0.124																	
July 31.....	....	0.014	0.065																	
August 1....	0.09	0.011	0.043	0.005										0.059	386	0.006		0.005	330	3 860
August 2....	0.07	0.009	0.032	0.022	0.004									0.067	355	0.005		0.006	390	4 380
August 3....	0.09	0.007	0.027	0.021	0.017	0.005								0.072	330	0.005		0.007	460	5 035
August 4....	0.05	0.007	0.022	0.011	0.016	0.022	0.003							0.077	336	0.005		0.007	460	5 300
August 5....	0.22	0.005	0.022	0.007	0.008	0.021	0.012	0.013						0.084	373	0.006		0.007	460	5 760
August 6....	0.09	0.005	0.016	0.005	0.006	0.011	0.012	0.053	0.005					0.113	367	0.006	0.0057 / 0.067 = 0.085	0.010	650	7 390
August 7....	0.03	0.003	0.016	0.005	0.004	0.007	0.006	0.051	0.022	0.002				0.116	380	0.006		0.010	650	7 590
August 8....	....	.....	0.011	0.004	0.004	0.005	0.004	0.026	0.021	0.007				0.082	615	0.009		0.007	460	5 360
August 9....	....	.....	.....	0.004	0.003	0.005	0.003	0.018	0.011	0.007				0.051	520	0.008		0.005	330	3 340
August 10...	....	.....	.....	0.003	0.003	0.004	0.003	0.013	0.007	0.004				0.037	380	0.006		0.003	200	2 420
August 11...	0.01	.....	.....	0.003	0.003	0.004	0.002	0.011	0.005	0.002	0.001			0.030	297	0.004		0.003	200	1 963
August 12...	0.06	.....	.....	0.002	0.002	0.003	0.002	0.009	0.005	0.002	0.002	0.004		0.031	275	0.004		0.003	200	2 030
August 13...	0.05	0.003	.....	.....	0.001	0.003	0.002	0.009	0.004	0.002	0.002	0.014		0.040	265	0.004		0.004	260	2 615
August 14...	0.82	0.012	0.049	.....	.....	0.002	0.002	0.007	0.004	0.001	0.001	0.014		0.092	292	0.005		0.005	330	6 020
August 15...	0.34	0.012	0.197	0.020	.....	.....	0.001	0.007	0.003	0.001	0.001	0.007		0.249	575	0.009		0.015	980	16 300
August 16...	0.40	0.006	0.189	0.082	0.024	.....	.....	0.004	0.003	0.001	0.001	0.005		0.315	1 030	0.016		0.019	1 240	20 620
August 17...	0.22	0.004	0.098	0.078	0.096	0.013	.....	.....	0.002	0.001	0.001	0.004		0.297	885	0.014	0.014 / 0.236 = 0.059	0.017	1 110	19 430
August 18...	0.32	0.003	0.066	0.041	0.092	0.053	0.019	.....	.....	.....	0.001	0.003		0.278	1 140	0.017		0.016	1 050	18 200
August 19...	0.08	0.003	0.049	0.027	0.048	0.051	0.077	0.005	.....	.....	.....	0.002		0.262	1 240	0.019		0.015	980	17 150
August 20...	....	0.002	0.041	0.020	0.032	0.026	0.074	0.019	.....	.....	.....	0.002		0.216	1 140	0.017		0.013	850	14 130
August 21...	0.43	0.002	0.033	0.017	0.024	0.018	0.038	0.018	0.026	.....	.....	0.002		0.178	838	0.013		0.011	720	11 650
August 22...	0.09	0.002	0.033	0.014	0.020	0.013	0.026	0.010	0.103	0.005	.....	0.002		0.228	1 080	0.017		0.025	1 639	14 910
August 23...	....	0.002	0.025	0.014	0.016	0.011	0.019	0.006	0.099	0.022	.....	0.001		0.215	1 650	0.025		0.023	1 506	14 070
August 24...	0.18	0.001	0.025	0.010	0.016	0.009	0.016	0.005	0.052	0.021	0.011	.....		0.166	1 360	0.021		0.018	1 189	10 860
August 25...	0.01	.....	0.016	0.010	0.012	0.009	0.013	0.004	0.034	0.011	0.043	0.001		0.153	932	0.014		0.017	1 110	10 010
August 26...	....	.....	.....	0.007	0.012	0.007	0.013	0.003	0.026	0.007	0.041	0.002		0.118	700	0.011		0.013	850	7 720
August 27...	0.01	0.001	.....	.....	0.008	0.007	0.010	0.003	0.022	0.005	0.022	0.002		0.080	575	0.009	0.0126 / 0.116 = 0.109	0.009	590	5 230
August 28...	....	0.002	.....	.....	.....	0.004	0.010	0.002	0.017	0.005	0.014	0.001		0.055	535	0.008		0.006	390	3 600
August 29...	0.08	0.002	0.005	.....	.....	.....	0.006	0.002	0.017	0.004	0.011	0.001		0.048	490	0.008		0.005	330	3 140
August 30...	....	0.001	0.019	.....	.....	.....	.....	0.002	0.013	0.004	0.009	0.001		0.049	433	0.007		0.005	330	3 205
August 31...	....	0.001	0.018	.....	.....	.....	.....	.....	0.013	0.003	0.007	0.001		0.043	386	0.006		0.005	330	2 813
September 1.	1.23	0.001	0.010	0.074	.....	.....	.....	.....	.....	0.003	0.007	0.001		0.105	373	0.006		0.021	1 370	6 870
September 2.	0.26	0.001	0.006	0.295	0.016	.....	.....	.....	.....	0.002	0.005	.....		0.325	475	0.007		0.064	4 180	21 270

in which, Q is the average 24-hr discharge and M, the area of the water-shed, in square miles. These values are listed in Column (16).

6.—The tabulation is divided into storm or peak periods, the divisions being easily identified in the column (Column (14)) of effective rainfall depths.

7.—Both rainfall and run-off are expressed in the same terms (that is, inches of depth). Within the storm period they bear a straight-line relationship to each other, and the coefficient is the ratio of their storm-period averages. These are to be found in Column (17).

8.—The coefficients are applied to the effective rainfall depths, I, computed values of D being obtained and recorded in Column (18).

9.—The computed values of D are converted into stream flow, in second-feet-days, and listed in Column (19).

10.—A graphic comparison of daily rainfall, effective rainfall depth, the observed hydrograph, and the computed hydrograph for the six water-sheds is finally plotted, as illustrated by Fig. 3 for the Licking River Water-Shed, and Fig. 4, for the Tuscarawas River Water-Shed.

Coefficients

The distribution graph is, in reality, a distribution coefficient, fulfilling that function in a manner much more in accord with the actual phenomenon than is the application of a single coefficient to the rainfall depth on the day in which it falls. The distribution coefficient breaks down rainfall depth and re-assembles it as it becomes effective to the flow of the stream. In this form there is a straight-line relationship between rainfall and run-off within the storm period, the slope of the curve becoming a coefficient that might be termed the "retention coefficient," because it is now called on to express the value of the losses to run-off only.

It will be seen that the retention coefficient marks the seasonal change in soil absorption. In addition, it is influenced by the degree of saturation immediately preceding the storm period and by the intensity of the storm itself. Beginning with a high value, throughout the storage period, January to March, it drops to a value of less than 0.10 through the growing season, May to September, increasing into the fall and winter months and reaching, in November, a value of approximately 0.50. When melting snow and ice are added to run-off, the value may exceed unity.

The seasonal fluctuation in the coefficient value shows marked stability over the entire region embraced by the study. It would seem that the way is opened for further refinement of the method by possible correlation of the coefficient with soil saturation in terms of previous rainfall, evaporation in terms of temperature, and with the average intensity of the storm which obviously affects the value of the coefficient.

The Pluviagraph

If the factor that is used to convert depth over the water-shed to stream flow, in second-feet-days, is applied to the effective rainfall depths, I, a

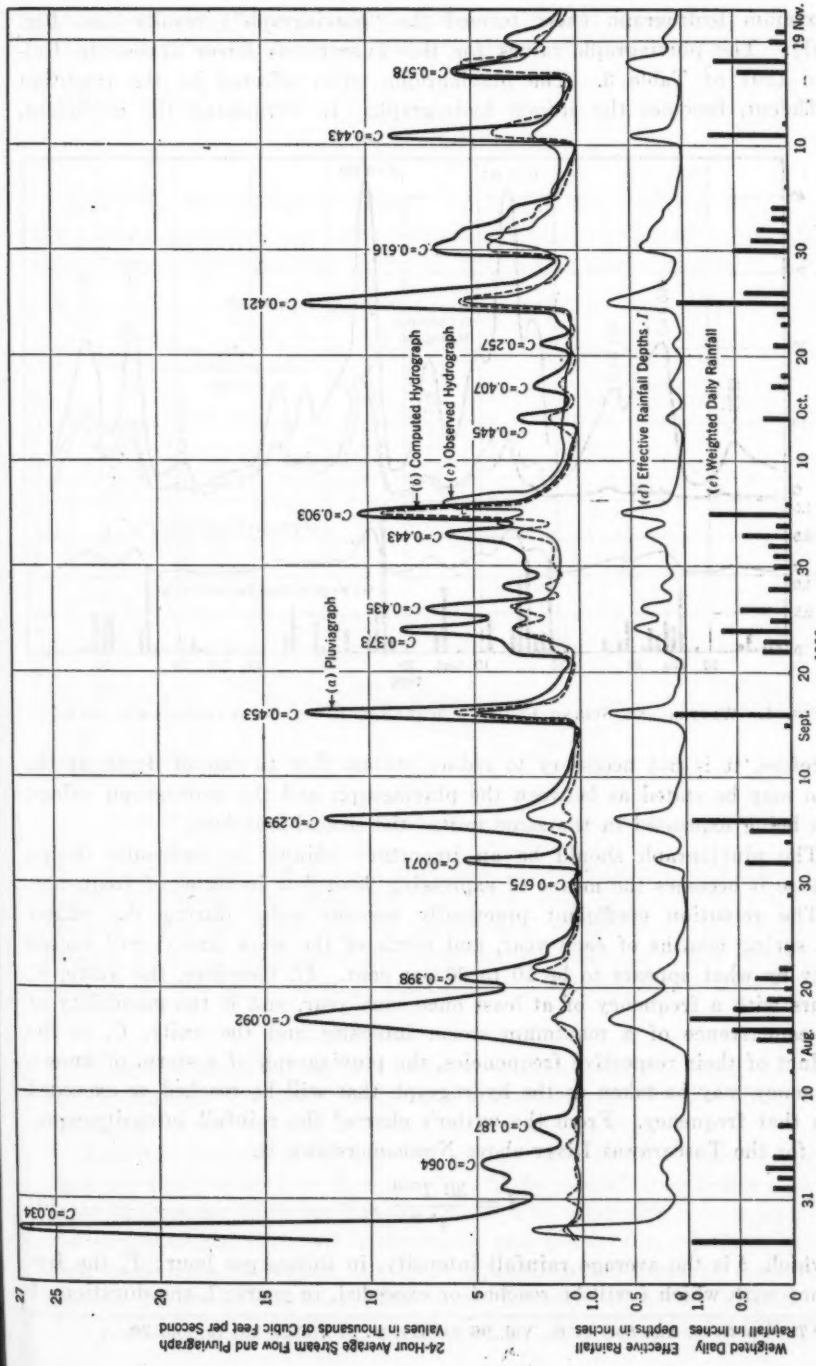

FIG. 3.—RAINFALL-STREAM FLOW CHARACTERISTICS, LICKING RIVER, AT TOBOSO, OHIO.

maximum hydrograph (here termed the "pluviagraph") results (see Fig. 3(*a*)). The pluviagraph values for the Tuscarawas River appear in Column (20) of Table 3. The pluviagraph, when affected by the retention coefficient, becomes the stream hydrograph. In developing the coefficient,

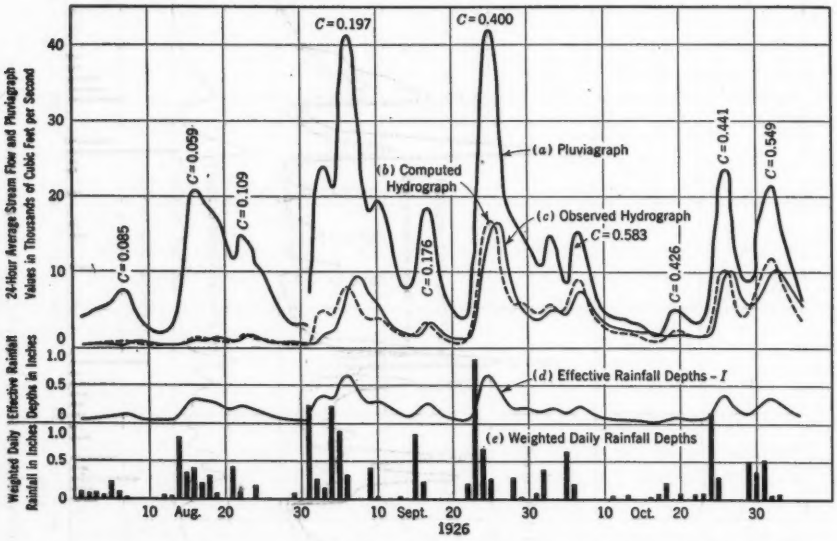

FIG. 4.—WINTER AND SPRING FLOW, TUSCARAWAS RIVER, NEWCOMERSTOWN, OHIO.

therefore, it is not necessary to reduce stream flow to run-off depth as the ratio may be stated as between the pluviagraph and the hydrograph values, both being expressed in the same units—the second-foot-days.

The pluviagraph should be an important adjunct to hydraulic design because it becomes the means of expressing flood flow in terms of frequency.

The retention coefficient practically reaches unity during the winter and spring months of each year, and north of the snow line it will exceed unity by what appears to be 10 to 20 per cent. If, therefore, the unity, C, occurs with a frequency of at least once each year, and if the possibility of the concurrence of a maximum storm intensity and the unity, C, is the product of their respective frequencies, the pluviagraph of a storm of known frequency may be taken as the hydrograph that will be reached or exceeded with that frequency. From the writer's charts,[8] the rainfall intensity equation for the Tuscarawas River above Newcomerstown is,

$$i = \frac{30\ T^{0.21}}{t^{0.80}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, i is the average rainfall intensity, in inches per hour; T, the frequency with which i will be reached or exceeded, in years; t, the duration of

[8] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 618, Figs. 18, 19, and 20.

rainfall, exceeding 1 hr, in minutes; K, a coefficient depending for value on locality; and x and n are exponents depending for value on locality.

When reduced by an area factor,[9] $(1 - 0.015 \sqrt[3]{A_m})$, Equation (3) becomes,

$$i = \frac{24\ T^{0.21}}{t^{0.80}} \qquad (4)$$

Reducing the average intensities of known frequency to daily rainfall depth and applying the distribution graph of the water-shed, results in the frequency-pluviagraph group shown in Fig. 5(*a*). These graphs are based on

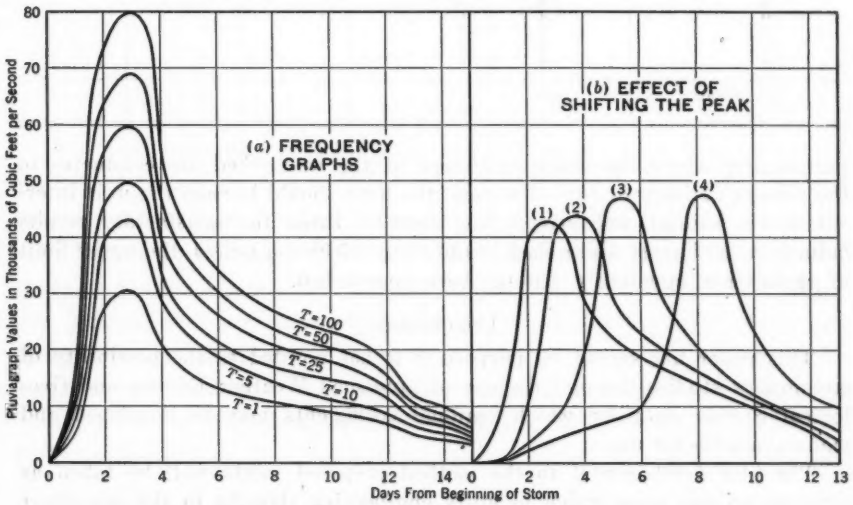

FIG. 5—PLUVIAGRAPHS, TUSCARAWAS RIVER, AT NEWCOMERSTOWN, OHIO

the assumption that the storm will parallel the rainfall intensity curve group, coming to a peak on the first day. That the storm curve may take any shape without materially affecting the pluviagraph is shown in Fig. 5(*b*), with Table 4, in which the peak is shifted to various positions in the storm period, without disturbing the average intensity or frequency of the storm.

LIMITATIONS OF THE METHOD

While the theory of the unit and distribution graphs appears to be generally applicable, the units used, and, therefore, the Weather Bureau and Geological Survey data, have their limits of application.

That the writer's study approaches the upper limit of value of U, and the lower limit of area, is shown in Fig. 3, the rainfall-stream-flow characteristics of Licking River at Toboso, Ohio. The relatively poor agreement of the computed and observed hydrographs and the erratic relationship of the peak coefficients, show that the 24-hr interval is too great to measure unit changes in stream discharge and storm intensity. For the smaller areas,

[9] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1150.

TABLE 4.—DISTRIBUTION FACTORS AND DAILY RAINFALL DEPTH CORRESPONDING TO FIG. 5(*b*)

Days	Distribution factors (effective percentage of total flow)	DAILY RAINFALL IN INCHES			
		Curve 1	Curve 2	Curve 3	Curve 4
1	6	2.40	0.12	0.17	0.12
2	24	0.36	2.40	0.24	0.13
3	23	0.24	0.36	0.36	0.14
4	12	0.17	0.24	2.40	0.17
5	8	0.14	0.17	0.14	0.24
6	6	0.13	0.14	0.13	0.36
7	5	0.12	0.13	0.12	2.40
8	4	....	....	....	....
9	4	....	....	....	....
10	3	....	....	....	....
11	3	....	....	....	....
12	2	....	....	....	....
Total	100	....	....	....	....

particularly where the shape and slope of the water-shed are conducive to the prompt delivery of run-off waters, the hour should become the unit interval of the rainfall and stream-flow record. From the satisfactory results gained on the largest water-shed in the study, (5 980 sq miles), the upper limit of application, apparently, has not been approached.

CONCLUSION

This paper has served its purpose if it has pointed to the possibility of stream-flow studies through the use of published Weather Bureau and Geological Survey data, by which regional coefficients may be developed and made available for use.

The idea fundamental to the method proposed might well be taken as common ground upon which to unify engineering thought in the important field of stream-flow determination.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DISCHARGE FORMULA AND TABLES FOR SHARP-CRESTED SUPPRESSED WEIRS

BY C. G. CLINE,[1] ESQ.

SYNOPSIS

The new discharge formula presented in this paper applies to sharp-crested weirs that occupy the full width of the channel; that is, weirs with the end contractions suppressed. This formula is entirely empirical and was developed by a mathematical study of the results of a series of weir experiments[2] made at Cornell University by E. W. Schoder, M. Am. Soc. C. E., and the late Kenneth B. Turner, Assistant Professor of Hydraulics. It differs from those in use at present in that a variable exponent, increasing with the head, replaces the customary fixed exponent (1.5), thus making it possible to obtain accurate discharges, even at low heads, by a comparatively simple form of coefficient. Furthermore, correction for the velocity of approach, which has been developed, depends entirely on the physical dimensions of the weir and can be applied directly.

Curves in this paper offer a comparison between the results of the Schoder and Turner experiments in comparison with the new formula and with the three standard formulas of Francis, Bazin, and Rehbock. These three standard formulas differ from one another, but each is supported by the experimental data over certain ranges of head and height of weir; however, over the entire range of head and height of weir covered by the experiments, the new formula agrees with the data more closely than any of these standard formulas. A set of complete tables for use in applying the new formula has been placed on record in Engineering Societies Library, 29 West 39th Street, New York, N. Y.[2a]

NOTE.—Discussion on this paper will be closed in April, 1934, *Proceedings*.

[1] Senior Asst. Engr., Water Power and Hydrometric Bureau, Dept. of the Interior, Canada, Niagara Falls, Ont., Canada.

[2] "Precise Weir Measurements," *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 999. (Reprints of this paper (No. 1711) are available.)

[2a] These tables may be purchased at a cost of fifty cents per set from the Secretary of the Society.

Status of Weir Formulas

The sharp-crested suppressed weir has been in use as a measuring device for many years, and a number of eminent hydraulic engineers have proposed formulas for computing its discharge. The paper by Professor Schoder and the late Professor Turner previously referred to, and the discussions thereon, together constitute an excellent review of the present status of this subject; the limitations of the various weir formulas in use are pointed out, with the net result that the question of a standard weir formula is left still unsettled. The weir experiments described in that paper comprise the most comprehensive set yet published, but the tentative formula proposed as an interpretation of these data does not seem convenient for practical use, since it requires such extensive measurements of the velocities in the approach channel as to constitute in themselves practically a measurement of the discharge. The formula presented in the present paper, based on part of the same data, can be used without any such auxiliary velocity measurements.

The paper by Professors Schoder and Turner contains a tabulation of the results of a large number of weir experiments. The set selected for this present analysis consists of Series D and O[3], in all of which the crest was a 12-in. brass plate. In making this selection, the intention was to have a set of accurate measurements all made under the same conditions except for the changes in the two variables, head and height of weir. Accordingly, all runs were omitted in which the natural flow of the water was disturbed by special fences; it was found necessary, also, to omit all those in which the head was less than 0.04 ft. In cases where two or more runs were made at practically the same head, mean values of head and discharge were used. Particulars as to height of weir, range of head, number of measurements,

TABLE 1.—Summary of Weir Experiments Selected

Schoder and Turner series	Height of weir, in feet	Range of head, in feet	Original number of volumetric measurements	Number selected after combining duplicates and omitting specials
E	0.5	0.03 to 2.0	57	26
N	0.75	0.01 to 2.3	100	31
F	1.0	0.03 to 2.4	91	30
G	1.5	0.02 to 2.4	90	29
H	2.0	0.02 to 2.5	87	28
I	3.0	0.02 to 2.7	93	29
J	4.0	0.02 to 2.7	93	29
K and O	5.5	0.01 to 2.7	174	54
D, L, and M	7.5	0.01 to 2.7	220	79
Totals	.....	.....	805	335

etc., taken mainly from the Schoder and Turner paper,[4] are listed in Table 1. All the weirs were 4.22 ft long, the full width of the channel, with no end contractions.

[3] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Appendices A and B, Tables 11, 12, 31, 32, 37, 38, 44, 45, and 55.

[4] *Loc. cit.*, Table 1, pp. 1000-1001.

Notation

In the metric formulas and tables of this paper, all lengths are in meters, and the discharges are in cubic meters per second. For purposes of conversion, note that 1 ft equals 0.3048 m. Unknown corrections (see Equation (10)) are expressed as *du*, *db*, etc. The following notation is introduced for the guidance of discussers:

a = a formula constant.
b = a formula constant (Equation (7)).
c = a constant coefficient (Equation (7)).
d = a substitution factor (Equation (7)).
f = a substition factor (Equation (19)).
g = acceleration due to gravity.
h = head (see, also, H); h_v = head corresponding to mean velocity of approach for use in Equation (2).
i = a substitution factor (Equation (18)).
k = a function of $P = \log a + K$ (Equation (8)).
n = a formula constant (Equation (7)).
r = a residual.
u = a formula constant.
v = a formula constant (Equation (16)).
x = a variable exponent of H.
C = a constant $= c\sqrt{2g}$ (Equation (9)).
H = head on weir; ϕH = dimensionless function, with single argument, H; $\phi(P\ H)$ = dimensionless function with two arguments, P and H (this constitutes the velocity of approach correction (see, also, (h)).
K = a function of $P = k - \log a$ (Equation (14)).
L = length of crest of weir.
P = height of weir; $\phi(P\ H)$ = dimensionless function, with two arguments, P and H (this constitutes the velocity of approach correction).
Q = discharge.
θ = a dimensionless multiplier.
ϕ = a symbol denoting function; ϕH = dimensionless function, with single argument, H; and $\phi(P\ H)$ = dimensionless function with two arguments, P and H (this constitutes the velocity of approach correction).

General Algebraic Form of Equation

With sufficient accurate experimental data available, the main difficulty in developing an accurate weir formula lies in selecting the proper algebraic form. Once this is done, it is comparatively simple, although sometimes somewhat laborious, to compute the best numerical values of the constants contained in the formula.

For a sharp-crested, suppressed weir, it is known from dimensional analysis that the equation for the discharge per foot of crest is of the form:

$$\frac{Q}{L} = \sqrt{2g}\, H^{\frac{3}{2}}\, \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, Q = total discharge of weir; L = length of crest; H = head; g = acceleration due to gravity; and θ = dimensionless multiplier.

Nearly all weir formulas are of this general type, the only difference being in the treatment of the term, θ, and in the "velocity of approach" correction. For purposes of comparison throughout this paper, three standard equations have been selected:

The Francis Formula:

$$\frac{Q}{L} = 3.33 \left\{ (H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The Bazin Formula:

$$\frac{Q}{L} = \left(0.405 + \frac{0.00984}{H} \right) \left\{ 1 + 0.55 \left(\frac{H}{H + P} \right)^2 \right\} \sqrt{2g}\, H^{\frac{3}{2}} \quad \ldots\ldots (3)$$

The Rehbock Formula:

$$\frac{Q}{L} = \frac{2}{3} \left(0.605 + \frac{1}{320\, H - 3} + 0.08\, \frac{H}{P} \right) \sqrt{2g}\, H^{\frac{3}{2}} \quad \ldots\ldots\ldots (4)$$

in which, h_v is the head corresponding to the mean velocity of approach.

In the experiments summarized in Table 1, changes in the discharge, Q, are due either to changes in the head, H, or to changes in the height, P, of the weir crest above the bottom of the approach channel. All other conditions remain unchanged. In Series E and M, $P = 0.5$ and 7.5 ft, respectively. Simultaneous values of $\log \frac{Q}{L}$ and $\log H$ for these two series are shown graphically in Fig. 1. The points for each series fall on a separate curve, which is practically a straight line for small values of H, but which shows definite curvature for large values. Series E, with the smaller value of P, has the greater discharge, the difference increasing as the head increases, so that its graph has a greater curvature.

A consideration of these two graphs, and of the principles of dimensional analysis, led to the conclusion that for the experiments summarized in Table 1, θ may be written as the product of two dimensionless functions, as follows:

$$\theta = \phi H \times \phi(P\, H) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, ϕH is a dimensionless function, with a single variable argument, H; and $\phi(P\, H)$, a dimensionless function, with two variable arguments, H and P. Using this expression for θ, Equation (1) may be written in the form:

$$\frac{Q}{L} = \phi H \times \sqrt{2g}\, H^{\frac{3}{2}} \times \phi\,(P\, H) \quad \ldots\ldots\ldots\ldots\ldots (6)$$

In the ordinary terminology of hydraulics, ϕH is the discharge coefficient, and $\phi(P\ H)$ is the "velocity of approach" correction, expressed as a separate multiplier. In the Francis formula, $\phi H = \frac{3.33}{\sqrt{2g}}$, which is constant for all

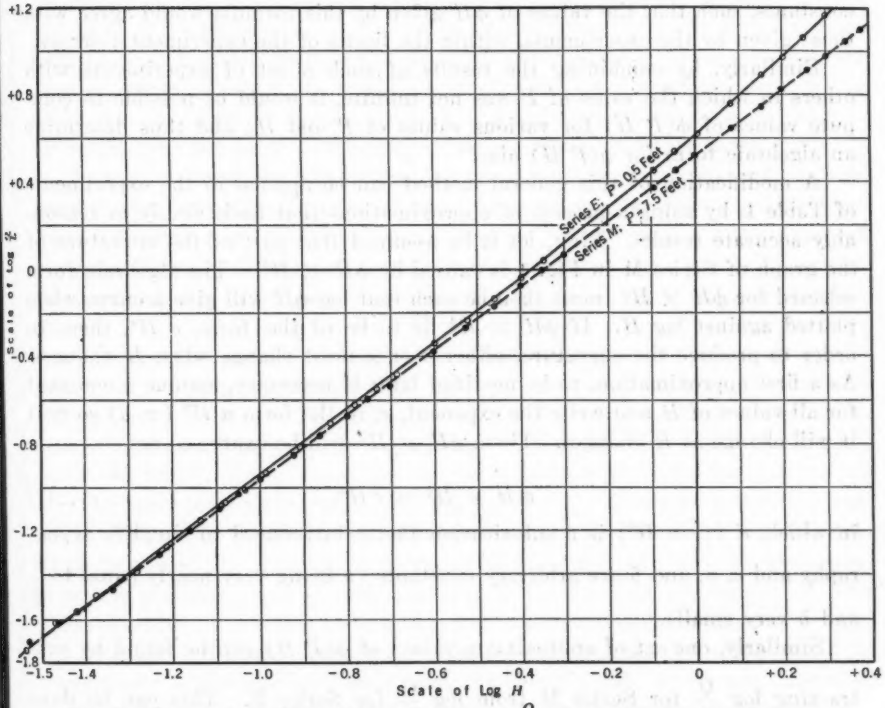

FIG. 1.—PLOTTING OF LOGARITHMIC VALUES OF $\frac{Q}{L}$ FROM SCHODER AND TURNER EXPERIMENTS, SERIES M AND E.

values of H, and the velocity of approach correction is applied by an indirect method involving a series of approximations. In the Bazin formula, $\phi H = \left(0.405 + \frac{0.00984}{H}\right)$ and $\phi\ (P\ H) = \left\{1 + 0.55\left(\frac{H}{H+P}\right)^2\right\}$. In the Rehbock formula, $\phi H \times \phi(P\ H) = \frac{2}{3}\left(0.605 + \frac{1}{320\ H - 3} + 0.08\ \frac{H}{P}\right)$. Evidently, the Bazin formula is the only one that is of the same form as Equation (6).

ALGEBRAIC FORM OF ϕH AND $\phi(P\ H)$

The algebraic forms proposed in this paper for ϕH and $\phi(P\ H)$ are different from those used in the Bazin formula. They are entirely empirical, having been derived mainly by studying the results of the Schoder and Turner experiments, and without much dependence on purely theoretical considerations.

If a set of experiments were available with an infinite height of weir, P, so that the velocity of approach could be neglected ($\phi(P\ H) = 1$), the only unknown in Equation (6) would be the coefficient, ϕH, and for each observed value of H the corresponding value of ϕH could be computed. It should then be possible to determine an algebraic form with suitable numerical constants, such that the values of ϕH given by this formula would agree with those given by the experiments, within the limits of the experimental errors.

Similarly, by combining the results of such a set of experiments with others in which the value of P was not infinite, it would be possible to compute values of $\phi(P\ H)$ for various values of P and H; and thus determine an algebraic form for $\phi(P\ H)$ also.

A modification of this general method can be applied to the experiments of Table 1, by using a process of approximations that leads finally to reasonably accurate results. First, let it be assumed that part of the curvature of the graph of Series M in Fig. 1 is caused by $\phi H \times H^{\frac{3}{2}}$. The algebraic form selected for $\phi H \times H^{\frac{3}{2}}$ must then be such that $\log \phi H$ will give a curve when plotted against $\log H$. If $\phi H \times H^{\frac{3}{2}}$ is to be of the form, $c\ H^x$, then, in order to produce the curvature, either c or x must change when H changes. As a first approximation, to be modified later if necessary, assume c constant for all values of H and write the exponent, x, in the form $n\ H^b\ (= d)$ so that it will change as H changes. Then, $\phi H \times H^{\frac{3}{2}}$ will be expressed as:

$$\phi H \times H^{\frac{3}{2}} = c\ H^d \qquad (7)$$

in which, $d\ (= n\ H^b)$ is a substitution factor introduced to simplify typography and c, n, and b are arbitrary constants (n being very nearly equal to $\frac{3}{2}$ and b very small).

Similarly, one set of approximate values of $\phi(P\ H)$ can be found by subtracting $\log \frac{Q}{L'}$ for Series M from $\log \frac{Q}{L}$ for Series E. This can be done directly, since there are a number of experiments in which the value of H is approximately the same in both series. When these values of $\log \phi(P\ H)$ are plotted against H, all the points fall close to a straight line through the origin. Evidently, $\log \phi(P\ H)$ can be written in the form:

$$\log \phi(P\ H) = kH \qquad (8)$$

in which, k is a constant when P is constant. The slope of the line gives an approximate value of k for Series E; and approximate values of k for the other series can be found in a similar manner.

Application of Method of Least Squares

The logarithmic equivalent of Equation (6) may now be written in the form:

$$\log \frac{Q}{L} = \log C + n H^b \log H + kH \qquad (9)$$

in which, $\log C = \log c \sqrt{2g}$ and $\log C$, n, b, and k are unknown constants, k having a different value for each different value of P. By adopting Equation (9) as a general equation for all the experiments listed in Table 1, and applying the method of least squares to the entire list, it was possible to secure additional information, as follows: (1) To determine more precise values of k for each value of P covered by the experiments, so that the relation between k and P may be expressed algebraically; (2) to determine more precise values of $\log C$, n, and b for use in a second solution; and (3) to show that this form of equation is adequate to represent the data of Table 1, by completing the solution, computing the residuals, and examining them.

In each solution by the method of least squares, the same general procedure was followed. The logarithmic form of equation was used so that each experiment would be of equal weight. Continuing the process of approximations, a differential formula was established (by partial differentiations of the original logarithmic formula), which showed the proportionate change in $\log Q$ (or in the logarithmic residuals), that would result from small changes in each of the constants. The residuals were found by substituting in the original logarithmic equation, assumed approximate values of the unknown constants. Each experiment in Table 1 furnished one residual, and each residual an observation equation, so that there were 335 observation equations in all. From these, the normal equations were formed and solved, and the most probable set of corrections was found. The application of these corrections to the approximate values assumed for the constants in the beginning, furnished the most probable values of the constants. In the final solution, the probable error of each constant in the formula was computed; the probable error of a single observation was also computed, or, more correctly, the probable difference between an observed and a computed discharge. As logarithmic equations were used, some of the probable errors are expressed as percentages.

The practical application of this process involved considerable work, especially in computing the residuals and in forming and solving the normal equations. The computations were arranged in tabular form and numerical checks were carried along with them. Adding and calculating machines, and tables of products, were used whenever possible.

This general method of utilizing the data of the experiments involves the assumption that all observations of head are exactly correct, and that any discrepancies that occur are due to errors in the discharge. This assumption, of course, is not strictly correct. A solution based on the assumption that errors of head and errors of discharge are equally probable, might be expected to give slightly different values for the constants, but it is doubtful whether such a refinement is warranted in this case.

In the first solution, the differential observation equation derived from Equation (9), was as follows:

$$d(\log C) + H^{(n)} \log H \, dn + 2.3026 \, n_1 H^{(n)} (\log H)^2 \, db + H \, dk = r_1 \ldots (10)$$

in which, d (log C), dn, db, and dk are the unknown corrections which, when applied to the approximate values used in Equation (11), will give the most probable values of the unknown constants. Common logarithms are used in Equation (10), as in all other formulas in this paper. The residuals, r_1, are found from the equation:

$$r_1 = \log \frac{Q}{L} - (\log C_1 + n_1 H^{(n_1)} \log H + kH) \ldots\ldots\ldots (11)$$

in which, $\log \frac{Q}{L}$, and H are given by the experiment and log C_1, n_1, ($b1$), and k are the assumed approximate values of the unknown constants. For the first solution, $n_1 = 1.5$, ($b1$) = 0.008, and the values used for k were those found by the graphical method described in the preceding section. The corrected values obtained for log C, n, and b were used in computing residuals for the second solution, the small size of these residuals showing the suitability of the algebraic form used. The corrected values of k, for the various

TABLE 2.—VALUES OF k FOR DIFFERENT VALUES OF P

Series	Height of weir, P	Value of k	Series	Height of weir, P	Value of k
E.....	0.5	0.0962	I.....	3.0	0.0154
N.....	0.75	0.0581	J.....	4.0	0.0123
F.....	1.0	0.0484	K and O.....	5.5	0.0116
G.....	1.5	0.0337	D, L, and M..	7.5	0.0105
H.....	2.0	0.0245	.....	.....	.....

values of P, are listed in Table 2, from which it may be noted that there is a definite relation between k and P, although a curve plotted from these data indicates certain minor inconsistencies.

FINAL ALGEBRAIC FORMS

In order to complete the weir formula, it is necessary to express, algebraically, the relation between the values of P and k in Table 2. For the smaller values of P, k decreases quickly as P increases, but, for the larger values, there is little change in k for relatively large changes in P. Furthermore, when the corresponding values of log k and log P are plotted, the resulting graph is not straight, but curved. From these considerations, and continuing the process of approximations by substituting trial values for certain of the constants and solving for the others, the following algebraic formula was finally adopted:

$$k = \log a + \frac{u}{1 + v P^2} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

By choosing suitable numerical values for the constants, Equation (12) can be made to agree well, although not perfectly, with the values of k in Table 2. That is the only reason for selecting this particular algebraic form, and it is quite possible that some other form might be found that would be more suitable.

Being a constant, $\log a$ is independent of P and, therefore, belongs in ϕH rather than in $\phi(P\ H)$, so that the final algebraic forms of ϕH and $\phi(P\ H)$ (see Equations (7) and (8)) are as follows:

$$\phi H = \frac{c\, a^H\, H^d}{H^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

and $\phi(P\ H) = 10^{KH}$, or $\log \phi(P\ H) = KH$, in which,

$$K = k - \log a = \frac{u}{1 + v\, P^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

These algebraic forms give reasonably accurate results, but there is no absolute certainty that they make an exactly accurate division between ϕH and $\phi(P\ H)$.

Final Least Squares Adjustment

A final adjustment of the entire equation was made by a second least squares solution. For this, Equation (9) was re-written in the form:

$$\log \frac{Q}{L} = \log C + n H^b \log H + H \log a + \left(\frac{u}{1 + v\, P^{\frac{1}{2}}}\right) H \ldots\ldots\ldots (15)$$

and the corresponding differential and residual equations are, as follows:

$$d\ (\log C) + H^{(b2)} \log H\ dn + 2.3026\ n_2\ H^{(b2)}\ (\log H)^2\ db + H\ d\ (\log a)$$
$$+ \left(\frac{1}{1 + v\, P^{\frac{1}{2}}}\right) H\ du - \left\{\frac{u\, P^{\frac{1}{2}}}{(1 + v\, P^{\frac{1}{2}})^2}\right\} H\ dv = r_2 \quad \ldots\ldots\ldots (16)$$

and,

$$r_2 = \log \frac{Q}{L} - (\log C_2 + n_2 H^{(b2)} \log H + H \log a + \frac{u H}{1 + v\, P^{\frac{1}{2}}}) \ldots\ldots (17)$$

in which, the values of $\log C_2$, n_2, and $(b2)$ were given by the first solution and the values of $\log a$, u, and v were those found by the trial method previously mentioned.

Numerical Values

The numerical values of the constants obtained in this final adjustment, together with their probable errors, are given in Table 3.

TABLE 3.—Numerical Constants in Weir Formula

Symbol	Numerical value	Logarithmic value	Probable error	Symbol	Numerical value	Logarithmic value	Probable error
$C = c\sqrt{2g}$.	3.2765	0.5154	±0.03%	b....	0.0081	.....	±0.00015
a.....	1.0195	0.0084	±0.03%	u....	0.2713	.....	±0.00014
n.....	1.5064	.....	±0.00027	v....	6.1452	.....	±0.00012

The complete formula, with the numerical values of the constants, may now be written in the logarithmic and numerical forms, as follows:

$$\log \frac{Q}{L} = 0.5154 + 0.0084\ H + 1.5064\ H^{0.0081} \log H + K H \ldots\ldots (18)$$

or,

$$\frac{Q}{L} = 3.276 \times (1.0195)^H \times H^f \times 10^{KH} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which, to simplify typography, $f = 1.5064H^{0.0021}$, and,

$$K = \frac{0.2713}{1 + 6.1452\ P^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Furthermore,

$$\sqrt{2g}\ \phi H = \frac{3.276 \times (1.0195)^H \times H^f}{H^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots (21)$$

and,

$$\phi(P\ H) = 10^{KH} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Table 4 has been prepared to show, for various values of the argument, H, the corresponding numerical values of the variable exponent, $f = 1.5064\ H^{0.0021}$,

TABLE 4.—NUMERICAL VALUES OF VARIABLE EXPONENT AND COEFFICIENTS

Head, H, in feet (1)	Equation (19)		Equations (21) and (6)
	Exponent, $1.5064H^{0.0021}$ (2)	Coefficient, $3.276\ (1.0195)^H$ (3)	Coefficient, $\sqrt{2g}\ \phi H$ (4)
0.1	1.478	3.282	3.449
0.2	1.487	3.289	3.360
0.3	1.492	3.295	3.327
0.4	1.495	3.302	3.316
0.5	1.498	3.308	3.313
0.6	1.500	3.314	3.314
0.7	1.502	3.321	3.318
0.8	1.504	3.327	3.324
0.9	1.505	3.334	3.332
1.0	1.506	3.340	3.340
1.2	1.509	3.353	3.359
1.4	1.510	3.366	3.379
1.6	1.512	3.378	3.399
1.8	1.514	3.392	3.420
2.0	1.515	3.406	3.441
2.5	1.517	3.439	3.495
3.0	1.520	3.472	3.549

and the variable coefficient, $3.276\ (1.0195)^H$, in Equation (19). This table also gives the coefficient $\sqrt{2g}\ \phi H$ from Equation (21), for use with the fixed exponent, 1.5, in Equation (6). The value of the variable exponent (Table 4) is 1.5 at 0.6 ft, 0.022 less than 1.5 at 0.1 ft, and 0.020 greater than 1.5 at 3.0 ft; so that the greatest differences from 1.5 are less than ± 1.5 per cent. However, this comparatively small change in the exponent makes a considerable difference in the corresponding coefficient, as can be seen by plotting Column (1), Table 4, against Column (4). The coefficient for use with the variable exponent (Column (3), Table 4) plots as a straight line, whereas the coefficient for use with the constant exponent, 1.5 (Column (4), Table 4) is a somewhat complicated curve, intersecting the straight line at the two points, $H = 1.0$ ft and $H = 0.6$ ft, practically a straight line above $H = 1.0$ ft, but with rapidly increasing curvature below $H = 0.6$ ft.

This peculiarity of the coefficient for use with the fixed exponent, 1.5, is one of the most serious difficulties in devising an accurate weir formula. It is neglected entirely in the Francis formula, probably because Francis did not make any experiments with heads less than 0.6 ft; but it is reproduced more or less accurately in the Bazin and Rehbock formulas by the introduction of a term containing H in the denominator.

The coefficient, $3.276\ (1.0195)^H$, varies as a straight line. When expanded by the binomial theorem, neglecting the third term and all subsequent terms, it becomes $3.276\ (1 + 0.0195H)$ which is the equation of a straight line. The value of the neglected third term is only 0.1% for $H = 3$ ft, and less for smaller values of H; so that for all practical purposes the two expressions are equal and may be used interchangeably in Equation (19). However, the original form is more convenient for use with logarithms.

Accuracy of New Formula

The probable difference between an observed and a computed weir discharge, as given by the final least squares solution, is ± 0.65 per cent. If the weir experiments could be regarded as exact, this would be the probable error of a single value of discharge computed by the formula, and its smallness shows that Equation (19) agrees very closely with the measurements. If the formula is regarded as exact, $\pm 0.65\%$ is the probable error of a single weir experiment, which is an indication of a high order of consistency in the experiments.

The agreement of the new formula (Equation (19)), with the weir experiments is shown graphically in Fig. 2, for three values of P. Similar agreement was found for intermediate values. In order to condense the figures as much as possible, the discharge per foot of crest, $\frac{Q}{L}$, has been divided by $H^{\frac{3}{2}}$, and the quotient for each experiment, plotted against the corresponding value of H, is shown by the circles.

The full line curves in Fig 2 represent the corresponding values given by Equation (19). All these individual curves have been assembled in Fig. 3. They constitute a "family" of curves that change shape and inclination to the H-axis when the value of P is changed. Each individual curve, in addition to fitting its own points as closely as possible, must also maintain a symmetrical position in Fig. 3. There are certain minor inconsistencies among the experiments themselves that make it impossible for any such symmetrical family of curves to fit all the points perfectly; this may be due partly to certain features of the weir equipment, which are discussed in another section of this paper. On that account, although the present set of curves differs from the experiments by more than 1% in some cases, it is felt that the search for an accurate weir formula has been carried as far as these experiments warrant. The method of least squares, of course, has resulted in the selection of the best numerical values of the coefficients in the particular type of formula proposed and any improvement could be made

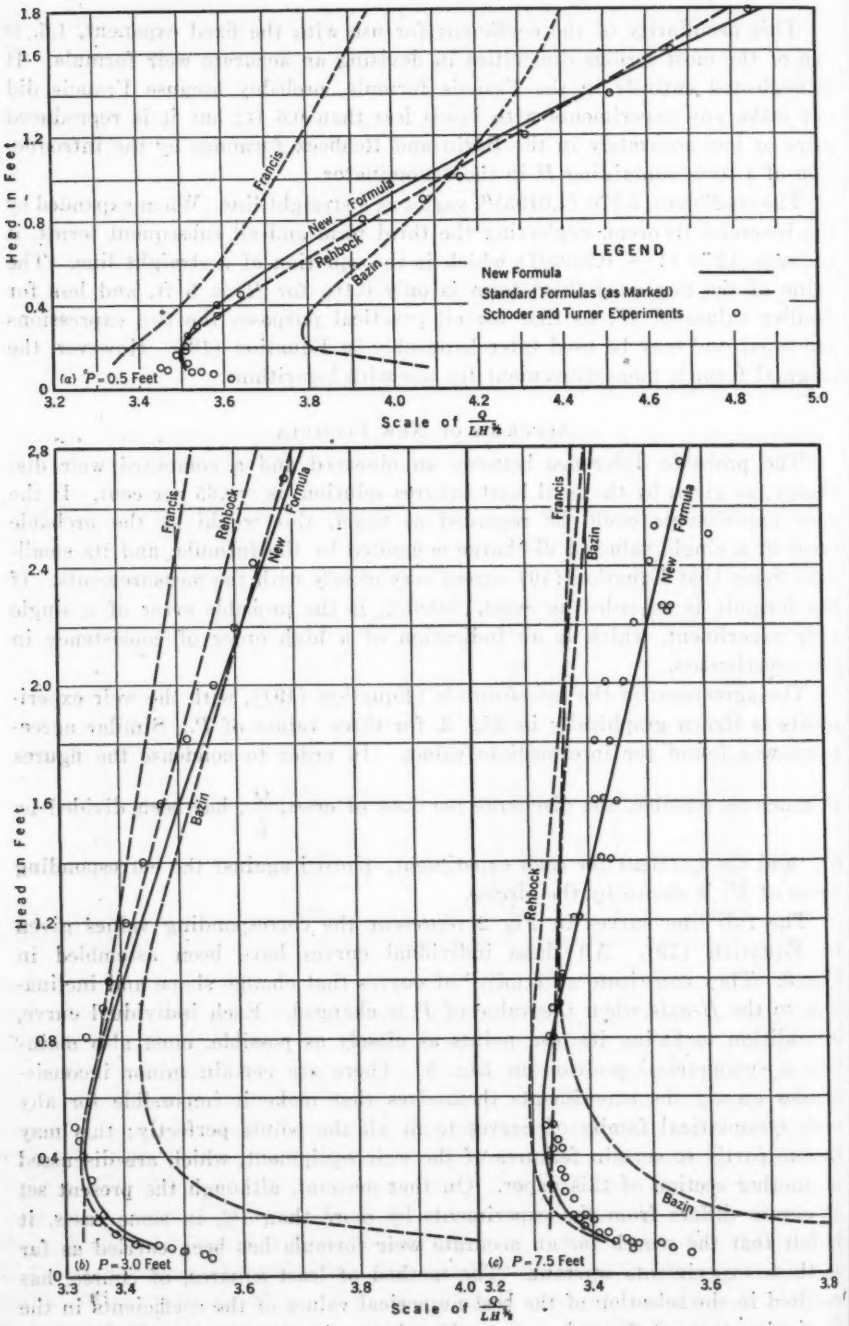

FIG. 2.—PLOTTED VALUES OF $\frac{Q}{LH^{\frac{3}{2}}}$ FROM SCHODER AND TURNER EXPERIMENTS FROM EQUATION (19) AND FROM THREE OTHER FORMULAS.

only by some change in the algebraic form of Equation (19). Additional experiments will be required to show whether this is necessary.

The broken lines in Fig. 2 show the values of $\frac{Q}{L H^{\frac{3}{2}}}$ as given by the three standard weir formulas, Equations (2), (3), and (4). The differences in their algebraic form allow them to agree with one another for certain

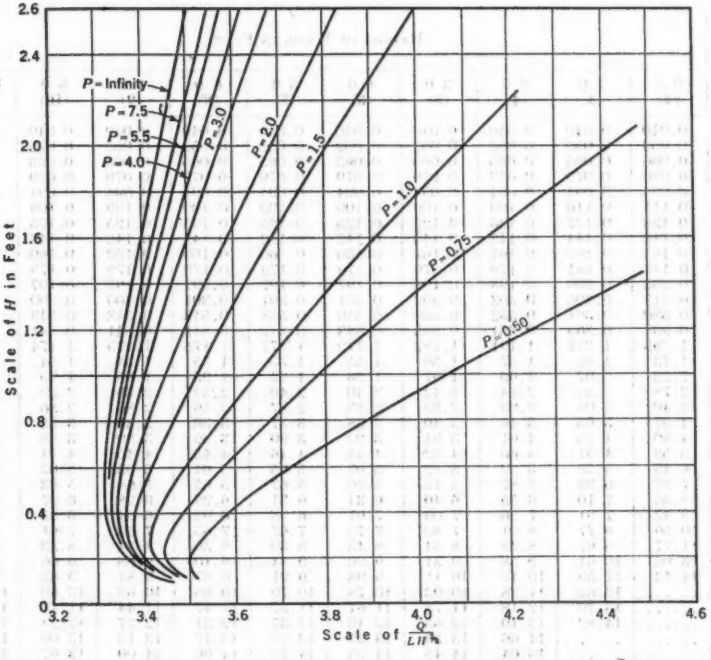

FIG. 3.—CURVES SHOWING RELATIONSHIP BETWEEN HEAD AND $\frac{Q}{L H^{\frac{3}{2}}}$ FOR VARIOUS VALUES OF P, FROM EQUATION (19).

values of H and P and to disagree for other values. For the full range of H and P covered by the Schoder and Turner experiments, none of these formulas agrees with the experiments as well as the new formula (Equation (19)) proposed in this paper. However, for values of H not greater than 2 ft and values of P not greater than 4 ft, the Rehbock formula does agree with the experiments fully as well as the new formula.

DISCHARGE TABLES

For practical field and laboratory use, the writer has prepared a discharge table by solving Equation (18) for values of H at intervals of 0.01 ft. It shows directly the discharge, in cubic feet per second per linear foot of crest, in terms of the two arguments: H, the head on the weir, and P, the height of the weir above the channel of approach. Table 5 is a condensed form of this discharge table, suitable for plotting curves. The table was confined to

the range of heads covered by the experiments, except that, for the higher weirs, it was extended to an even 3 ft, although the experiments go only to 2.7 ft. Column (11), Table 5, shows the discharge computed for a weir of infinite height; that is, with no velocity of approach.

TABLE 5.—DISCHARGE FOR SUPPRESSED WEIRS, IN CUBIC FEET PER SECOND PER FOOT OF CREST LENGTH

Head, in feet	HEIGHT OF WEIR, IN FEET									
(1)	0.5 (2)	1.0 (3)	2.0 (4)	3.0 (5)	4.0 (6)	5.0 (7)	6.0 (8)	7.0 (9)	8.0 (10)	Infinity (11)
0.05...	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06...	0.053	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
0.07...	0.066	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065
0.08...	0.080	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079	0.079
0.09...	0.095	0.094	0.094	0.094	0.094	0.094	0.094	0.094	0.094	0.094
0.10...	0.111	0.110	0.109	0.109	0.109	0.109	0.109	0.109	0.109	0.109
0.11...	0.128	0.127	0.126	0.126	0.125	0.125	0.125	0.125	0.125	0.125
0.12...	0.146	0.144	0.143	0.143	0.142	0.142	0.142	0.142	0.142	0.142
0.13...	0.164	0.162	0.161	0.161	0.160	0.160	0.160	0.160	0.160	0.160
0.14...	0.183	0.181	0.179	0.179	0.179	0.179	0.179	0.179	0.178	0.178
0.15...	0.203	0.200	0.198	0.198	0.197	0.197	0.197	0.197	0.197	0.197
0.20...	0.313	0.306	0.303	0.302	0.301	0.301	0.301	0.300	0.300	0.300
0.30...	0.580	0.561	0.552	0.550	0.549	0.548	0.548	0.548	0.548	0.547
0.40...	0.908	0.869	0.851	0.845	0.843	0.842	0.841	0.841	0.841	0.839
0.50...	1.293	1.224	1.191	1.182	1.179	1.177	1.175	1.175	1.174	1.171
0.60...	1.73	1.62	1.57	1.56	1.55	1.55	1.55	1.54	1.54	1.54
0.70...	2.23	2.07	1.99	1.97	1.96	1.95	1.95	1.95	1.95	1.95
0.80...	2.78	2.55	2.44	2.42	2.40	2.40	2.39	2.39	2.39	2.38
0.90...	3.40	3.08	2.93	2.89	2.88	2.87	2.86	2.86	2.86	2.85
1.00...	4.07	3.65	3.46	3.40	3.38	3.37	3.36	3.36	3.35	3.34
1.10...	4.80	4.25	4.01	3.94	3.92	3.90	3.89	3.89	3.88	3.86
1.20...	5.59	4.91	4.60	4.52	4.48	4.46	4.45	4.45	4.44	4.42
1.30...	6.45	5.59	5.22	5.12	5.07	5.05	5.04	5.03	5.02	4.99
1.40...	7.37	6.33	5.87	5.75	5.70	5.67	5.65	5.64	5.63	5.60
1.50...	8.36	7.10	6.55	6.40	6.34	6.31	6.29	6.28	6.27	6.22
1.60...	9.42	7.91	7.26	7.09	7.01	6.98	6.95	6.94	6.93	6.88
1.70...	10.56	8.77	8.01	7.80	7.72	7.67	7.65	7.63	7.61	7.56
1.80...	11.77	9.67	8.78	8.54	8.45	8.39	8.36	8.34	8.33	8.26
1.90...	13.06	10.61	9.58	9.31	9.20	9.14	9.10	9.08	9.06	9.98
2.00...	14.43	11.59	10.42	10.11	9.98	9.91	9.87	9.84	9.82	9.73
2.10...	.....	12.62	11.28	10.93	10.78	10.70	10.66	10.62	10.60	10.50
2.20...	.....	13.70	12.18	11.78	11.61	11.52	11.47	11.44	11.41	11.30
2.30...	.....	14.82	13.10	12.65	12.46	12.37	12.31	12.27	12.24	12.12
2.40...	.....	.....	14.06	13.56	13.35	13.24	13.17	13.13	13.09	12.95
2.50...	.....	.....	15.04	14.48	14.25	14.13	14.06	14.00	13.97	13.81
2.60...	.....	.....	.....	15.44	15.18	15.04	14.96	14.90	14.87	14.70
2.70...	.....	.....	.....	16.42	16.14	15.99	15.89	15.84	15.79	15.60
2.80...	.....	.....	.....	17.43	17.11	16.95	16.85	16.78	16.74	16.53
2.90...	.....	.....	.....	18.46	18.12	17.94	17.82	17.75	17.70	17.48
3.00...	.....	.....	.....	19.51	19.14	18.94	18.82	18.74	18.68	18.44

Table 6 is a selected list of K-values computed by Equation (20), for values of P from 0.5 to 8.0 ft. A complete table, for intervals of $P = 0.1$ ft, is necessary, but for purposes of demonstration the trends indicated by Table 6 must suffice. Tables such as Table 4 (Column (4)) and Table 6, may be used for computing discharges independently of Table 5 by means of the following formula:

$$\frac{Q}{L} = \sqrt{2g}\,\phi\, H \times H^{\frac{1}{2}} \times 10^{KH} \qquad (23)$$

This method is useful for computing the discharge for low weirs with heights intermediate between those listed in the discharge table, in which simple interpolation between the columns will not give the correct discharge

for high heads. Tables in metric units similar to Table 4 (Column (4)) and Table 6, in English units are necessary in some computations and have been prepared by the writer.

TABLE 6.—SELECTED VALUES OF K BY EQUATION (20), IN ENGLISH UNITS, FOR USE IN EQUATION (23)

P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K	P, in feet	K
0.5....	0.0855	0.8	0.0502	1.2	0.0299	1.8	0.0171	3.0	0.0082	6.0	0.0030
0.6....	0.0703	0.9	0.0434	1.4	0.0243	2.0	0.0148	4.0	0.0054	7.0	0.0024
0.7....	0.0590	1.0	0.0380	1.6	0.0202	2.5	0.0107	5.0	0.0039	8.0	0.0019

WEIR DESIGN

Strictly speaking, the discharge tables offered for discussion in this paper should be used only with weirs constructed and arranged similarly to that on which the experiments were performed, a detailed description of which is given in the paper by Professors Schoder and Turner. The weir plate was of rolled brass, 12 in. wide, with the top edge machine-beveled and with the up-stream face smooth. The head was measured, at a section 11.74 ft up stream from the crest, by direct measurement in the channel, with a steel tape and also by a gauge in a stilling-well. This distance of 11.74 ft is greater than the 6 ft used by Francis and less than the 16.4 ft used by Bazin.

Back-water must be avoided, of course; the jet must be fully aerated, and should not cling to the weir plate. At very low heads, a violation of this latter condition may introduce erratic results; on that account, the discharge table was not carried below 0.05 ft.

In the Schoder and Turner experiments, the water was supplied by a 30-in. pipe which emptied into the weir structure about 25 ft from the crest and only 5 ft below it. It is not the intention in this paper to suggest the duplication of this supply arrangement, which presumably was necessitated by local conditions, but rather to draw attention to the suggestions[5] of Professor Th. Rehbock that: (*a*) The approach channel should be of sufficient length and of the same width throughout; (*b*) the water should enter the approach channel axially; and (*c*) its free approach to the weir should not be disturbed by built-in structures. This provides a natural arrangement that can be duplicated readily.

The Schoder and Turner experiments, in which the approach conditions differed most from conditions suggested herein, namely, those in which special fences were used, were specifically excluded from consideration in this paper, but apparently some of the experiments that were used have been affected somewhat by other features of the special approach arrangements. Those with high heads and consequent high velocities of approach would be affected more than those with low heads and low velocities. If the experiments for high heads were omitted, the curves could be made to fit the remaining points better (see Fig. 2). Professor Rehbock's contention that the position

[5] *Transactions*, Am. Soc. C. E., Vol. 93 (1929) p. 1143.

of the supply pipe must have interfered with the free flow of water for weirs 5.5 and 7.5 ft high, may account for the fact that, for certain heads, the discharge is greater for the 7.5-ft weir than for the 4-ft weir.

However, these inconsistencies are really of small magnitude (possibly of the order of $\pm 1\%$), and their mention here should not detract seriously from the merit of the experiments; nor should they obscure the fact that the new formula agrees with the entire set of experiments with a probable error of only ± 0.65 per cent.

Standard Discharge Tables

A standard set of discharge tables for suppressed weirs would be of great practical value. Having such tables, the practical man need not concern himself with the value of the discharge coefficient, the method of applying the velocity of approach correction, nor indeed with any particulars of the formula. The only question he need ask is whether the discharges given by the tables are accurate; the remainder he can leave to the computer who prepares the tables.

The discharge table, of which Table 5 is a sample, is submitted as a tentative discharge table for standard weirs with the approach conditions suggested by Professor Rehbock as stated previously, even if it is based on the Schoder and Turner experiments, which were made with somewhat different approach arrangements. It is probable that the table is not seriously in error from this cause, but it would be an advantage to have it extended to include higher heads; also, additional experiments would possibly allow a more precise determination of the formula. Accordingly, a program is suggested by means of which this tentative table might be extended and if necessary corrected, as follows: (1) Adopt a standard design of suppressed weir; (2) using weirs of this standard design, make a set of precise weir experiments, similar to those made by Professors Schoder and Turner; (3) select an algebraic form of equation that will be sufficiently flexible to agree with the results of the experiments over the entire range of all the variables introduced into the experiments; (4) use the method of least squares to determine the best values of the numerical constants in the formula; and (5) use the formula to compute a set of discharge tables covering the entire range of all the variables.

Changing to Metric Units

Discharges in metric units may be found from Table 5 by converting the head from meters to feet and the discharge from cubic feet per second to cubic meters per second, or liters per second. However, it would be more convenient to use a discharge table in metric units, and such a table can be computed by solving Equation (23), or its logarithmic equivalent, together with the values of $\sqrt{2\,g}\,\phi H$ and of K, which have been computed for the metric system and are on record in Engineering Societies Library as stated previously.

The discharge formula and the auxiliary terms, with the numerical values of the coefficients for the metric system, are given in the "Summary."

Summary

The general type of discharge formula proposed in this paper for sharp-crested suppressed weirs (see Equation (6)) is as follows:

$$\frac{Q}{L} = \sqrt{2g}\,\phi H \times H^{\frac{3}{2}} \times \phi\,(P\,H) \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which (see Equation (13)), $\phi H = \dfrac{c \times a^H \times H^d}{H^{\frac{3}{2}}}$; $\phi\,(P\,H) = 10^{KH}$;

and (see Equation (14)), $K = \dfrac{u}{1 + v\,P^{\frac{3}{2}}}$. Whence,

$$\frac{Q}{L} = C \times a^H \times H^d \times 10^{KH} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which, $C = c\sqrt{2g}$.

Inserting the numerical values of the coefficients for the two systems of units, Equation (25) becomes in English units (feet and cubic feet per second):

$$\frac{Q}{L} = 3.276 \times (1.0195)^H \times H^f \times 10^{KH} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and (see Equations (18) and (19)),

$$\log \frac{Q}{L} = 0.5154 + 0.0084\,H + 1.5064\,H^{0.0081} \log H + KH \quad \ldots\ldots (27)$$

in which, $K = \dfrac{0.2713}{1 + 6.1452\,P^{\frac{3}{2}}}$; and $f = 1.5064\,H^{0.0081}$. Furthermore (see Equation (21)):

$$\sqrt{2g}\,\phi H = \frac{3.276 \times (1.0195)^H \times H^f}{H^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

In metric units, Equation (25) becomes,

$$\frac{Q}{L} = 0.3044 \times (1.0654)^H \times (3.2808\,H)^i \times 10^{KH} \quad \ldots\ldots\ldots\ldots (29)$$

and,

$$\log \frac{Q}{L} = 1.4834 + 0.0275\,H + 1.5210\,H^{0.0081}\,(\log H + 0.5160) + KH \ldots (30)$$

in which, $K = \dfrac{0.890}{1 + 36.52\,P^{\frac{3}{2}}}$; and $i = 1.5210\,H^{0.0081}$. Furthermore,

$$\sqrt{2g}\,\phi H = \frac{0.3044 \times (1.0654)^H \times (3.2808\,H)^i}{H^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots (31)$$

The complete discharge table at intervals of $H = 0.01$ ft is not published, a condensed form (Table 5) being considered adequate for purposes of discussion. It shows directly the discharge, in cubic feet per second per linear foot of crest, in terms of the two arguments, both expressed in feet, namely,

H, the head on the weir; and P, the height of the weir above the bottom of the channel of approach. A table compiled for intervals of $H = 0.01$ ft is probably sufficiently accurate for most practical purposes, but it has been shown how the results of future experiments may be used to make small corrections in the table, if necessary, and to extend it for higher heads.

Tabular values of $\sqrt{2g}\ \phi H$ and of K, are contained in Tables 4 and 6. Tables such as these, compiled for intervals of $H = 0.01$ ft and $P = 0.1$ ft, respectively, may be used in Equation (23) for computing discharges independently of the discharge table, or to supplement that table.

The adoption of a standard design for sharp-crested suppressed weirs and the preparation of an elaborate and accurate set of standard discharge tables, as advocated in this paper, should help to restore the lost prestige of the weir as an accurate device for measuring water.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

RENEWAL OF MITER-GATE BEARINGS, PANAMA CANAL

BY CLINTON MORSE,[1] JUN. AM. SOC. C. E.

SYNOPSIS

The equipment and methods of raising and moving twelve miter-gate leaves of the Panama Canal, at Miraflores, are described in this paper. Considerable wear on the vertical bearing surfaces made replacement advisable. The quoin-bearings could be made accessible only by raising the leaves clear of the pintles and moving them from the wall. The largest leaves are 82 ft high and have an estimated gross weight of 790 tons.

Comparisons are made between the experimental equipment and the methods used at Pedro Miguel, in 1929, and the present system. Improvements in technique, suggested by experience gained in raising twenty-two leaves during three over-haul periods, are emphasized.

INTRODUCTION

The Panama Canal was opened to commercial traffic on August 15, 1914. Three lifts are required to reach the summit level, which is normally 85 ft above sea level. Twin chambers, each 110 ft wide, with a usable length of 1 000 ft, permit simultaneous northbound and southbound lockages. A diagram of the Miraflores Locks is shown in Fig. 1. At the time of their construction the lock-gates were by far the largest and heaviest that had ever been built. They contained a number of new features that have been incorporated in later designs. An unusual device was the use of removable nickel-steel plates for the miter and quoin-bearings shown in Fig. 2.

The leaves pivot on hemi-spherical nickel-steel pintles, 16 in. in diameter (Fig. 3). On the gates at the sea end of the lower chambers at Miraflores and on the lower guard-gates at Gatun, the pintle bushings—twelve in all—are of carbon steel; the others are of manganese bronze. Each leaf is held

NOTE.—Discussion on this paper will be closed in April, 1934, *Proceedings*.

[1] Jun. Engr., The Panama Canal, Balboa Heights, Canal Zone.

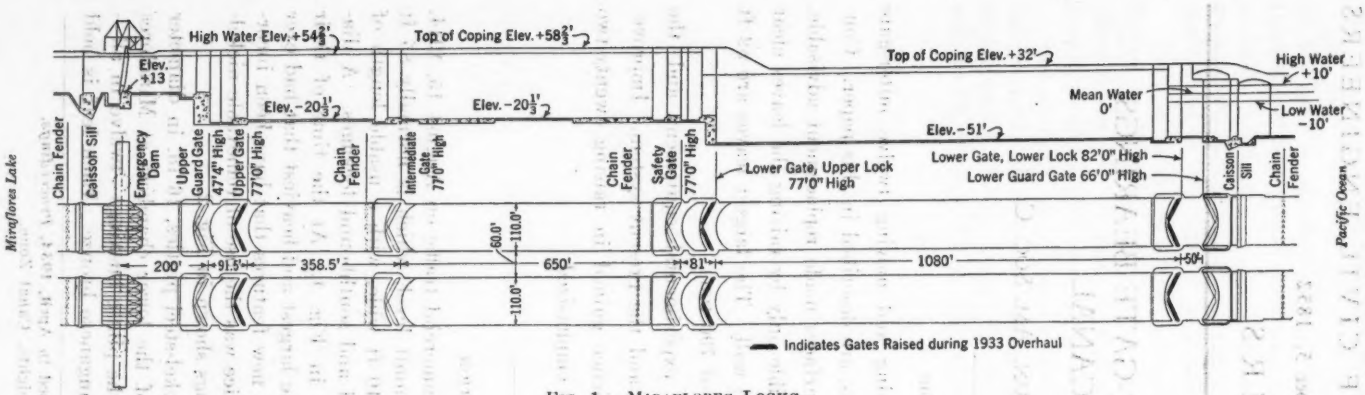

FIG. 1.—MIRAFLORES LOCKS.

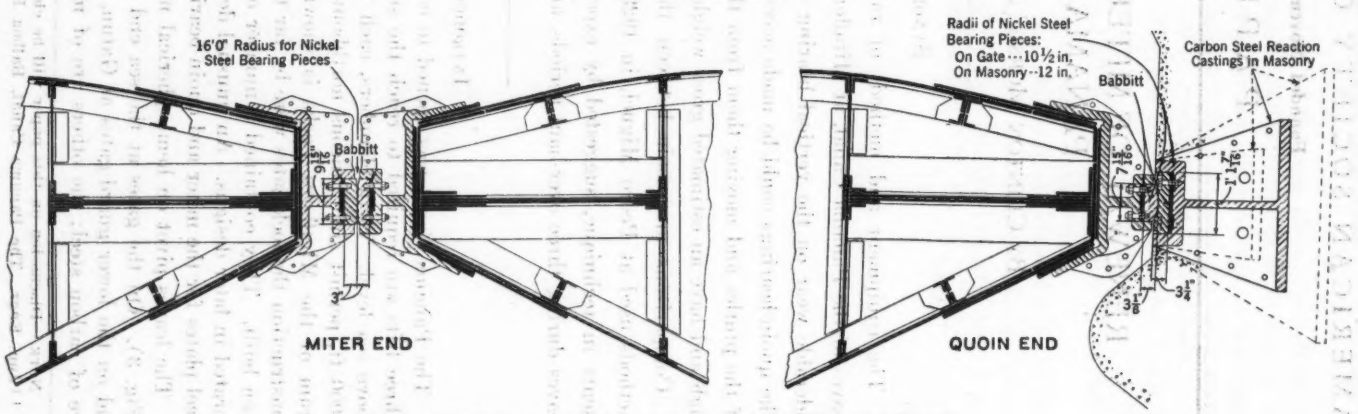

FIG. 2.—SECTION SHOWING MITER AND QUOIN BEARING-PLATES

at the top, by a pin connection, to a V-shaped, vanadium-steel yoke which, in turn, is keyed to anchorages deeply embedded in the concrete walls.

The major dimensions of the gates (of which there are 92) are given in Fig. 4. Their gross weight is 426 to 790 tons each. The design and erection of these gates have been described[2] concisely by Henry Goldmark, M. Am. Soc. C. E., who was in direct charge of the work.

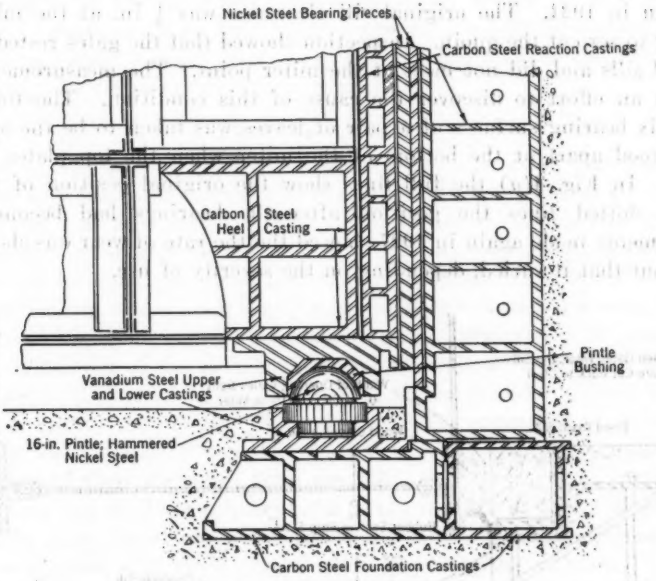

FIG. 3.—PINTLE AND QUOIN-BEARINGS.

The locks are overhauled during the dry season, which starts about January 1 and ends early in May. The first over-haul periods involved mostly repainting the gates and replacing the wooden sills damaged by the teredo.

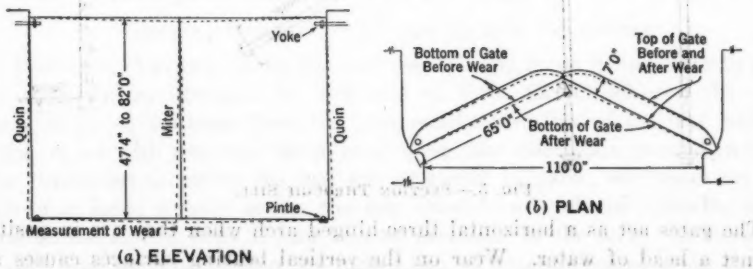

FIG. 4.

In the period between 1924 and 1929, extensive repairs were made to the rising stem and the cylindrical valves.[3]

[2] "The Panama Canal," *Transactions*, International Eng. Congress, Vol. II, pp. 87-129.
[3] "Lock Valves," Manuals of Eng. Practice, No. 3, Am. Soc. C. E., pp. 54-71.

Present policy requires that each lock be given a general overhauling every four years, the periods alternating between the Atlantic and Pacific locks. Thus, one chamber of the canal is out of service, for about three months during the dry season, once every two years.

Effects of Worn Bearings

The wear on the bearing-plates was first measured during the over-haul at Gatun in 1924. The original sill clearance was $\frac{1}{2}$ in. at the miter end, tapering to zero at the quoin. Inspection showed that the gates rested against the fixed sills and did not meet at the miter point. The measurements were made in an effort to discover the cause of this condition. The total wear on the six bearing surfaces of a pair of leaves was taken to be the space the leaves stood apart at the bottom of the miter when the top plates were in contact. In Fig. 4(*a*) the full lines show the original position of the gate and the dotted lines the position after the bearings had become worn. Measurements made again in 1927 showed the the rate of wear was about $\frac{3}{32}$ in. per yr, but that it varied, depending on the severity of use.

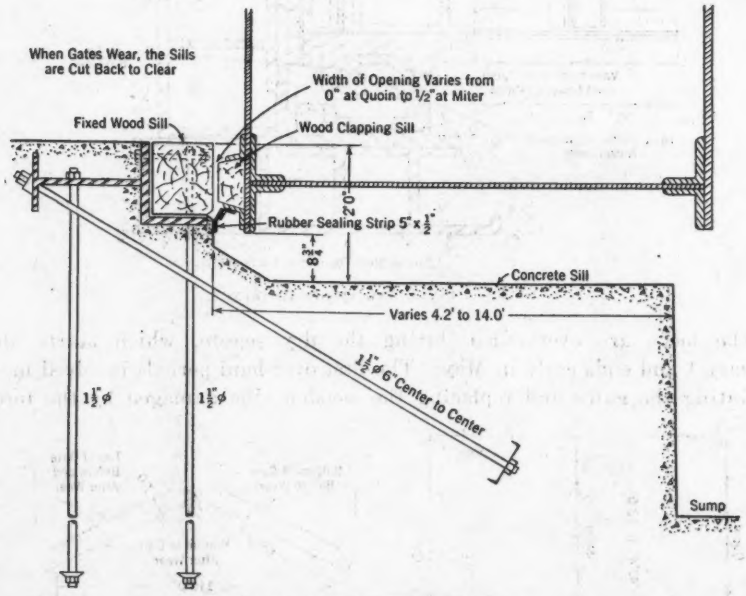
When Gates Wear, the Sills are Cut Back to Clear
Width of Opening Varies from 0" at Quoin to ½" at Miter
Fixed Wood Sill
Wood Clapping Sill
1'-0"
Rubber Sealing Strip 5" x ½"
8¾"
Concrete Sill
Varies 4.2' to 14.0'
1½" ø 6" Center to Center
1½" ø
1½" ø
Sump

Fig. 5.—Section Through Sill.

The gates act as a horizontal three-hinged arch when they are in position against a head of water. Wear on the vertical bearing surfaces causes rib-shortening and a consequent down-stream movement of the miter point. The movement down stream is 1.10 in. per in. of wear. It is necessary to trim the wood sills periodically (Fig. 5) in order to restore the original clearance.

The result of this trimming has been to increase the tendency of the leaves to warp when in operation. The top of the miter, which showed no wear,

remained in its original position, whereas the bottom was forced down stream until the bearing-plates made contact; this caused the leaves to bend along a diagonal line extending from the top of the miter to the pintle. The distortion of the gates is indicated in Fig. 4 (*b*).

The objectionable results of the wear on the bearing surfaces made it obvious that they should be renewed, beginning with the main operating gates and those showing the greatest wear. While the nominal life of the bearing-plates is twenty years, increasing traffic would shorten their life. Twenty-two years would be required to renew all the bearing surfaces if eight leaves were repaired every two years. Even this plan would require that twelve leaves be repaired during one over-haul.

The Trial Raising

The only way in which the quoin-bearing could be made accessible was to raise the leaf from the pintle and then move it from the wall. Since no attempt had ever been made to raise and move leaves of this size before, the first attempt was largely an experiment to determine the proper equipment and methods to be used for renewing all the bearings. Economy demanded, and the short dry season made imperative, the design of standard equipment and methods which function with production-line efficiency. A proposal to float the gates out of the chamber and to repair them in dry dock was abandoned and E. S. Randolph, M. Am. Soc. C. E., developed a method of raising the leaves within the chamber by hydraulic jacks connected to needle-beams under the leaf, in such a way that the leaves are raised only enough to clear the pintle and then are rolled from the wall.

The first test of this equipment was made in 1929 at Pedro Miguel, when two leaves were raised and replaced successfully. Desirable minor changes in the equipment and mode of procedure were obvious at once. Two years later, eight leaves were raised at Gatun. The speed and efficiency with which the equipment functioned during this over-haul removed all doubts as to the practicability of the adopted design. Additional equipment was purchased and twelve leaves (see Fig. 1) were raised and replaced at Miraflores between January 3 and April 15, 1933.

General Description of the Raising Equipment

The small clearance under the leaf (see Fig. 5) made it necessary to place the twelve jacks (grouped in four sets of three jacks each) on the sides. Each set is equi-distant from the transverse center line of the leaf, and the jacks are set with the rams down, bearing against steel slabs grouted in place. The needle-beams under the leaf are arranged in pairs, one beam on each side of a jack; a yoke across the top (closed end) of the cylinder passes through the slotted ends of link bars on each side of the jack; the lower end of each link bar fits between the forked end of a needle-beam; a pin through the fork and link bar completes the connection; and the weight of the leaf is carried on its flange-like bottom edge. Details of the jack-and-needle-beam assembly are shown in Fig. 6, and a typical set of three jacks is shown in Fig. 7.

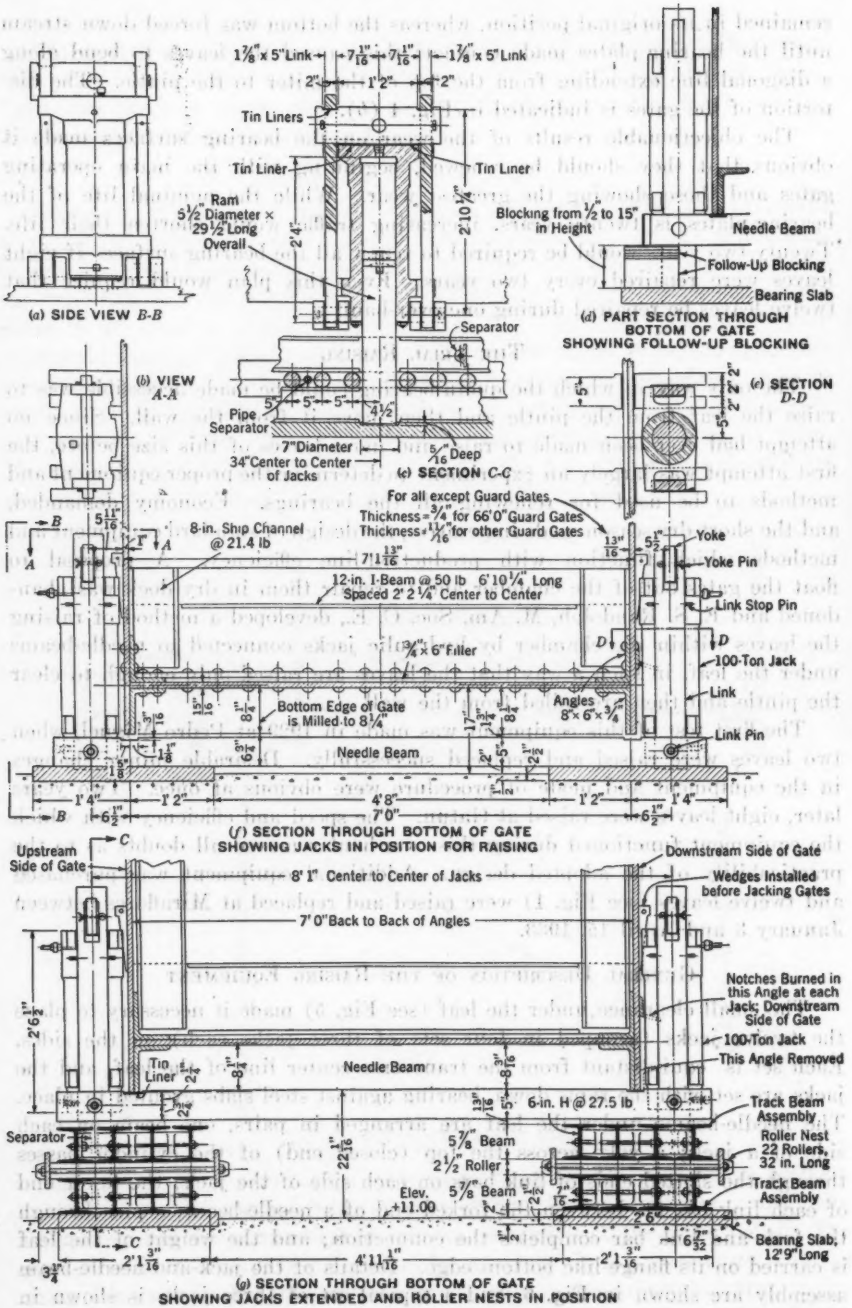

FIG. 6.—ARRANGEMENT OF JACKS

A mixture of switch oil and transformer oil is the hydraulic medium used to operate the jacks, and pressure is supplied by an electrically driven pump mounted on a detachable platform on the down-stream side of the leaf. The twelve jacks are interconnected to this pumping unit.

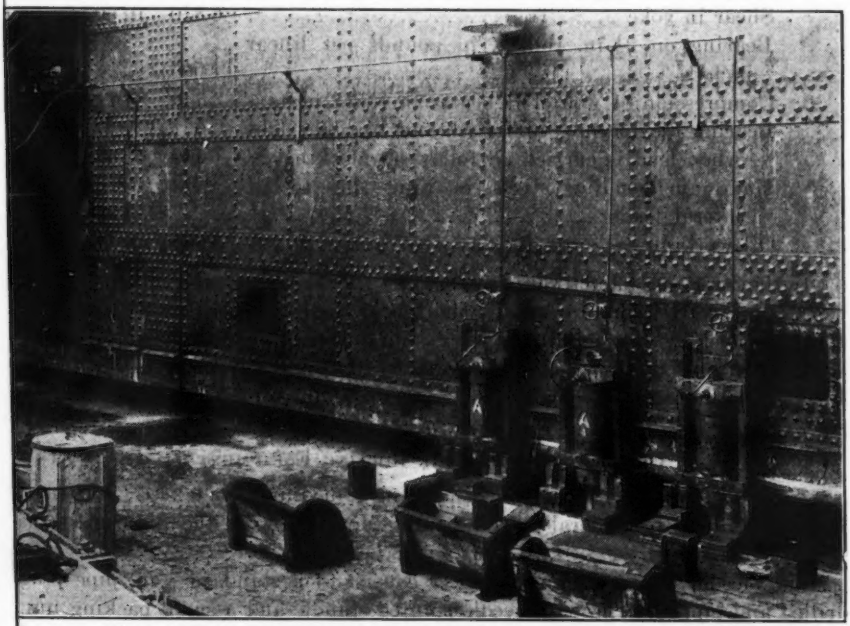

FIG. 7.—VIEW OF JACKS AND PUMP PLATFORM

A set of upper and lower roller tracks and a train of rollers is provided at each group of jacks. The needle-beams bear across the top of the upper tracks, with the lower tracks resting on the bearing slabs and separated from the upper tracks by the roller train. The track and roller assembly is designed to permit 4 ft of travel. Fig. 6(*c*) and Fig. 6 (*g*) show the leaf raised and the tracks and rollers in place. Overturning is prevented by a structural steel guide-frame and four steel cables anchored in the lock-walls.

Some of the principal unit stresses (in pounds per square inch) applying to this problem are, as follows:

	Normal	Maximum
Bearing between bottom of leaf and needle-beam	7 050	10 575
Extreme Fiber Stress in Needle-Beam When:		
Lifted by jacks	20 600	30 900
Resting on shims	26 000	40 350
Resting on tracks	8 000	12 000
Shearing stress in pins	8 300	12 450
Bearing in pins	15 700	23 550
Bending in pins	30 000	45 000

	Normal	Maximum
Tension in link at pin-hole	13 000	19 500
Bearing at top of slot	21 100	30 150
Double shear below pin-hole	8 290	12 425
Extreme fiber stress in yoke	18 850	28 275
Shear in yoke	6 870	10 505
Bearing on $2\frac{1}{2}$-in. rollers, in pounds per linear inch	(1 000)	.....
Compression of web of H-beams in roller nests	3 140	4 710
Bearing on masonry under edge of bearing slabs when load is carried on follow-up blocking	400	600
Maximum bending stress in rams due to all lateral loads	.....	9 220
Oil working pressure in jacks	5 600	8 400

In the foregoing tabulation normal stresses are those caused by all parts functioning perfectly with an even distribution of the load; maximum stresses are those derived by the conditions of design. Due to the high unit stresses anticipated and the lack of clearance for enlarging the sections, the yokes, links, link-pins, and needle-beams were made of forged nickel-manganese steel.

Details of Lifting and Rolling Equipment

Jack Cylinders.—The jack cylinders are medium grade steel castings, made according to Federal Specifications No. 170, with the dimensions as shown in Fig. 6(*c*).

The bleeder valves shown in Fig. 7 consist of a standard $\frac{1}{2}$-in. pipe plug drilled and tapped for a $\frac{1}{4}$-in. needle-pointed spindle and a small packing nut. An oil column made up of a glass tube and $\frac{1}{4}$-in. brass pipe fittings is screwed into the side of the plug. The needle-point controls the outlet and allows the mixture of oil and air to pass up through the vertical glass tube and thence through a downward curving spout. A bucket is held under the spout to catch the oil expelled with the air. The absence of air bubbles passing through the glass tube gives an accurate indication as to when all the air has been expelled.

A gag screw, placed near the open end of the cylinder, can be tightened against a Tobin bronze gag plug to prevent the ram from falling out during handling. A felt wiping ring is fastened around the open end of the cylinder to prevent small particles of grit from working up between the cylinder and the ram.

Jack Rams.—The rams (see Fig. 6(*c*)) are of rolled carbon-steel bar stock (Society of Automotive Engineers Standard No. 2315) ground to a diameter 0.008 in. less than the bore of the cylinder, and case-hardened No attempt was made to hold the diameters of the rams and cylinders to an exact dimension, but each ram was required to fit its own cylinder with the specified clearance.

The original jacks were provided with rams 0.005 in. smaller than the bore and were not case-hardened. The cylinder did not have the felt wiping ring and,

consequently, small particles of grit entered when the jacks were lowered. When one of these small particles was caught it would raise a small sliver of metal on the ram. As the ram was pushed out, this sliver would gradually roll into a ball and "freeze" the jack. The jacks functioned perfectly at Miraflores and not a single case of scored rams or cylinders was evident.

Pumping System.—Pressure is supplied by a vertical triplex pump having a $\frac{1}{2}$-in. bore and a 4-in. stroke. The driving force is a 5-hp motor acting through a reduction gear giving a speed of 37 rpm. The unit is compactly mounted on a roofed platform which is hooked to the down-stream side of the leaf (a part of this platform is visible in Fig. 7). The high-pressure lines lead through calibrated gauges and flexible couplings to pipe lines, fastened to the leaf and leading to each end; thence they lead through the leaf to the up-stream side. Tee connections and short lengths of pipe lead from the main pressure line on the leaf to stop-valves, and thence through flexible couplings to the individual jacks. The stop-valves at the jacks (see Fig. 7) are normally open and are used only in testing or for cutting out individual jacks in case of their failure.

The pipes on the platform are arranged so that all valves are within reach of one man. The miter end, which is the lighter, tends to raise faster than the quoin so that it is necessary to throttle, slightly, the valves controlling the miter end. A spirit-level fastened to the gate enables the operator to raise both ends at the same rate, approximately 0.3 in. per min. The jacks are interconnected so that no jack can carry more than one-twelfth of the total load.

The high-pressure line is an extra heavy steel pipe, $\frac{1}{2}$ in. in inside diameter, with forged fittings and especially built valves, all of which were subjected to a pressure inspection test before acceptance. The threads were machine-cut on the job, in order to insure a perfect fit and a thick solution of shellac was found to be the most satisfactory medium for sealing the threaded joints. The flexible couplings are of annealed copper tubing, $\frac{1}{4}$ in. in inside diameter, with brass nipples soldered to the ends. Annealed copper washers were used in all couplings and unions because they had proved more satisfactory than either leather or tin, and little trouble was experienced with leaking connections.

One pump platform and one set of jacks serves two leaves. The jacks are uncoupled and assembled on the other leaf of the pair as soon as one leaf has been raised, but the pump platform is left in place until the leaf has been pushed from the wall. In transferring the platform to the other leaf, it is necessary to uncouple only the flexible joint at the edge and thus leave the fixed piping in place. The original pumping unit was placed on the floor of the sump on the up-stream side of the gates. The maze of pipes interfered considerably with the assembly of the rolling equipment. The present unit is compact, out of the way, and functions perfectly.

Shims.—Banca tin shims were used at all points of bearing in the lifting system, to overcome any uneven bearing and to insure an even distribution of the load by crushing. They were placed between the jacks and the yokes,

between the yokes and the link bars, and between the needle-beams and the gate. Their effectiveness was thoroughly demonstrated, some being crushed to one-half their original thickness, while others were only slightly dented.

Banca tin was selected after a number of compression tests had also been made on solid and perforated specimens of aluminum, copper, lead, and type metal.

Roller Tracks.—The roller tracks are centered under the edge of the leaf, which necessitates assembling them around the extended jack rams. To permit this assembly, the inside flanges of the outer beams and the outside flanges of the inner beams are cut back to the web in three places. The upper and lower tracks are alike except for the location of these holes for the jack rams. Hand holes are cut through the webs of the outer beams at each jack location. These holes are necessary to permit the removal and replacement of the rocker disks under the rams. The three inner beams are bolted together in a unit so that the outer beams can be slipped into place quickly. The four upper tracks are made to move as a unit by connecting the up-stream and down-stream tracks to each other, and then connecting the quoin and miter ends by two H-beams.

The original tracks were not provided with hand holes and were only loosely bolted together. The difficulty experienced in assembling the original tracks was overcome by bolting the inner beams together and providing separate bolts for the outer beams.

Bearing-Blocks.—For the lifting and rolling operations the leaves are fixed in a partly open position, so as to allow sufficient room for scaffolds on the miter ends of the leaves after they had been pushed from the wall. In this position, each leaf partly overhangs the sump. A pair of bearing-blocks (made up of plates and angles) was provided under the miter end of each leaf to overcome the difference in elevation. One block in each pair has one corner cut off diagonally, and is slightly shorter, so that it may be placed parallel to the line of rolling and against the arched sill. A leveling screw is placed at each corner of the blocks making it a simple matter to bring them quickly and easily to grade. Punch-marks in the top plates indicate the longitudinal and transverse center lines of the jacks. The larger block of the pair weighs 14 980 lb; the smaller, 12 630 lb.

The original blocks were steel boxes set in place and filled with concrete, which made them so heavy that they were extremely difficult to remove with the available equipment. These old blocks were "scrapped" and the new all-steel blocks are easily placed in approximate position, later to be removed, by two electric cranes.

Bearing Slabs.—A rolled-steel bearing slab, (see Fig. 6(*g*)) is placed at each group of jacks. Three counterbores (Fig. 6(*c*)) are cut in the top of each slab to serve as recesses for the rocker disks under the jack rams. Eight leveling screws, one on each corner, and four along the center line serve to keep the slab level while it is being grouted in place. The center lines of the jacks are indicated by punch-marks. The two slabs at the quoin end rest directly on the concrete sill; and the up-stream miter slab is sup-

ported entirely on the bearing-block, but the down-stream slab bears partly on the bearing-block and partly on the sill.

Guide-Frame and Cables.—The guide-frame shown in Fig. 8 is provided as a safety measure against possible overturning. It is designed in accordance with the specifications for steel railway bridges of the American Railway Engineering Association. The wall end of the frame is pinned to a casting

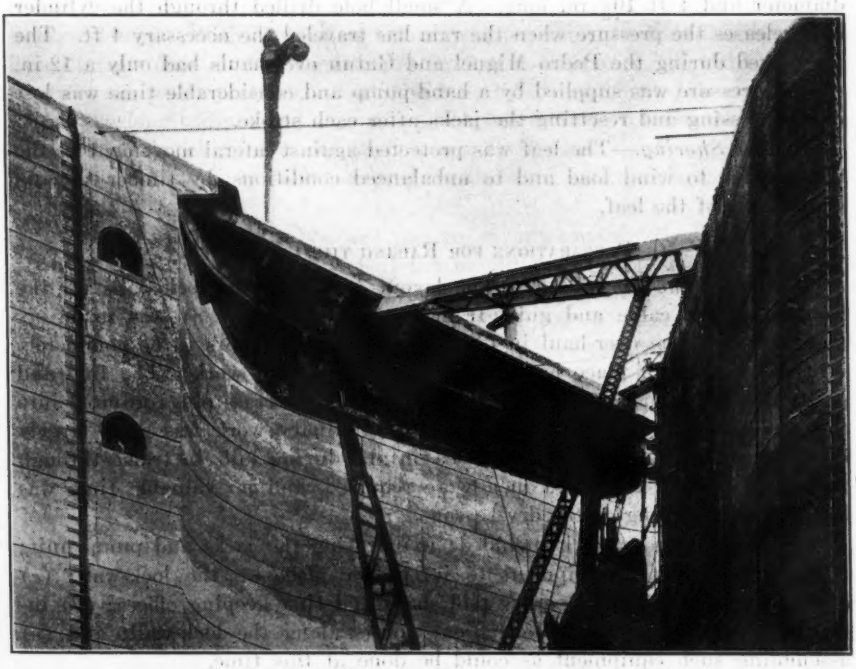

FIG. 8.—GUIDE-FRAME TO PREVENT OVERTURNING.

that is held in place by a large jack-bolt and a 4-in. pin through a corbel of the leaf recess cover. The other end, equipped with rollers, rests on the leaf and is provided with a 4-ft slot to permit the leaf to travel as it is pushed from the wall. A casting that holds a spherical-headed pin is bolted to the top center of the leaf. This pin is provided with a square nut, made in halves and bolted together around the spherical head. The nut travels in the guide-frame slot when the leaf is pushed from the wall.

Four $1\frac{1}{4}$-in., galvanized, extra strong, plow-steel cables are fastened to the leaf with turnbuckles and anchored in the lock walls, two up stream and two down stream (see Fig. 8). These cables are used to correct slight lateral deflections during raising, lowering, and moving, the guide-frame being depended upon in case of emergency.

Pusher Jacks.—The leaves are pushed from the wall by two hydraulic jacks, one on each side of the leaf, which are suspended from the H-beams that connect the upper miter tracks to the upper quoin tracks. These jacks

are interconnected, and pressure is supplied by the same pumping unit used in raising the leaf. The jacks, with rams permitting a 4-ft stroke, roll the leaf the required distance with one continuous movement.

The jack cylinder is a 5-ft section of 3-in., double, extra heavy, lap-welded, steel pipe. It is closed at one end by a plug and a bearing head welded in place. The bore is 2.425 in. in diameter with a solid steel ram, 2.410 in. in diameter and 4 ft 10½ in. long. A small hole drilled through the cylinder wall releases the pressure when the ram has traveled the necessary 4 ft. The jacks used during the Pedro Miguel and Gatun over-hauls had only a 12-in. stroke; pressure was supplied by a hand-pump and considerable time was lost in compressing and resetting the jacks after each stroke.

Timber Shoring.—The leaf was protected against lateral movement at the bottom, due to wind load and to unbalanced conditions, by timber shoring at the ends of the leaf.

Preparations for Raising the Leaves

Layout on the Lock-Walls.—The layout on top of the lock-walls for the location of the cable and guide-frame anchors (see Fig. 8) was made six months before the over-haul in order to allow the anchors to be completely set and grouted in place before beginning the actual work. The line and position of each anchor were referenced carefully to brass plugs grouted into the concrete and center-punched. This preliminary work was done largely by the operating forces in the wet season and during lulls in traffic through the Canal. Very few extra men were required, and no valuable time was lost at the beginning of the dry season.

Other preparatory work included: (*a*) Testing the jacks and pump unit; (*b*) distributing the equipment to its proper place on the lock-wall; (*c*) erecting and stocking temporary tool sheds and time-keeping offices; (*d*) installing field telephones at convenient points along the lock-wall; and (*e*) assembling such equipment as could be done at this time.

The equipment was distributed in an orderly manner by assigning a letter and a color to each pair of leaves. Each piece of equipment was stenciled with its proper mark as soon as it was received. Two electric cranes, converted from towing locomotives, and capable of ascending or descending the ramps, were used for much of this distribution work.

Layout on the Chamber Floor.—As soon as the weather indicated the beginning of the dry season, a floating caisson was placed in position and sunk at low tide. The water remaining in the chambers (approximately 10 ft in the upper lock and 41 ft in the lower lock) was pumped out by four electrically driven centrifugal pumps in the caisson. While the chambers were being unwatered, elevators were placed in position and small jib cranes were erected on the tops of the leaves at the miter and quoin ends.

Work began in the upper chamber as soon as it was dry. The sills were cleaned of mud and the water was pumped out of the sumps. The clapping-sill on the leaf and its lower retaining angle (see Fig. 5) were removed and the bottom and lower part of the leaf scraped and cleaned.

The down-stream face of the leaf served as the base line for the location of the bearing-blocks and slabs. The correct position of the base line was located by swinging the gate to the position fixed for the jacking and rolling operations. This position is determined by a direct measurement from a brass plug that indicates the center line of the guide-frame anchor, to the center of the spherical-headed pin on the gate. The position of the leaf is checked carefully because a mistake in this measurement would delay the work seriously. With the leaf in the correct position, the base line is established on the floor of the chamber and the leaf is then swung to the full open (against the wall) position. With the base line established, brass plugs are set on the longitudinal and transverse jack center lines for reference points. The area to be chipped under the bearing-blocks and slabs is next marked out, and the location of each jack and needle-beam indicated by punch-marks on the leaf.

Certain rivets must be burned out to permit passing the high-pressure piping through the leaf and to bolt the pump platform hangers (Fig. 7) in place. Sections of the sill angle are burned back to allow the jacks to be placed close to the face of the leaf (see Fig. 6(*g*)). The sections of the sill to be "burned" are marked with white paint; those cut from the angle are saved and later welded into place.

The bottom edge of the leaf is milled at each needle-beam location to insure a perfect bearing between the beams and leaf. A horizontal reference line, for the milling machine, is marked on the leaf from a bench-mark established in the sump.

The bearing slabs (Fig. 6(*g*)) are set with their tops only slightly above the level of the concrete sill, making it necessary to chip a recess in the sill for each of the quoin slabs. Because the depths of the sumps vary in each lock, it is necessary to build a concrete mat under the bearing-blocks in order to bring them to their proper elevation. As soon as the leaf layout has been completed, workers begin chipping the areas under the blocks and slabs. After the blocks have been placed in correct position and elevation, the mat is formed by pouring cement mortar between the bottom of the block and the floor of the sump. The mortar consists of 1 bag of cement to 2 sacks of sand, with $2\frac{1}{2}$ lb of calcium chloride for each bag of cement. Three vertical 2-in. pipes through the blocks acted as vents and prevented the formation of air pockets.

The four bearing slabs were carefully leveled (the permissible variation being $\frac{1}{64}$ in.) and checked before and after they were grouted in place. The correct position of the slabs and blocks was easily established by bringing the punch-marks on them to piano wires stretched between the reference plugs.

Assembling and Testing Equipment.—As soon as the slabs were set and the milling was completed, the leaf was swung to the jacking position and the clearance between the milled bottom of the leaf and the slabs was carefully checked. A difference of $\frac{1}{16}$ in. in level would put undue stress in the equipment with the possibility of failure due to: (1) Crushing the edge of

the leaf; (2) bending a needle-beam; or (3) crushing the webs of the roller-track beams. The few slight differences were corrected by spot-welding thin steel shims to the bottom of the leaf (see Fig. 6(*g*)). The needle-beams were then placed and connected to the jacks, the pump platform was hung in place and connected to the fixed piping, and the timber shoring set in place. The guide-frame was next placed in position, the cables were attached, and the leaf-operating strut was removed. At this point the jacking equipment was ready for test.

Each pair of jacks on the same needle-beam was tested separately, the pressure being raised gradually to 105 tons per jack, which was equal to a 50% over-load. After this test, each group of three jacks (Fig. 7) was tested to the same pressure, during which levels were taken on the slab under the jacks. Occasionally, a deflection of $\frac{1}{64}$ in. would be recorded with a load of 315 tons on the slab, but in most cases no deflection could be observed. Any leaking connections were tightened and the valve stems repacked if necessary.

The next step was to pull the yoke-pin at the top of the leaf. The jacks were used to lift the miter end slightly which allowed the pin to slip out easily. Then the wedges that key the yoke to the anchorages were removed; some, being very tight, were dislodged with difficulty. Several methods were tried, but the most satisfactory proved to be the following: A pneumatic paving breaker with a square-end bit was held on the wedges until the vibrations had broken the rust film between them and heavy driving wedges and follow-up blocking were then placed back of the key wedges. The driving wedges were struck with the hammer of a minature pile-driving rig operated by the jib crane on the quoin end of the leaf. The necessary effect was secured by allowing a 150-lb weight to fall approximately 6 ft inside a 4-in. pipe placed over the head of the driving wedge. Ten or twelve blows of this hammer were usually sufficient to start the key wedges. While these wedges were being driven, two-thirds of the weight of the leaf was carried by the jacks and the remainder by follow-up blocking under the ends of the needle-beams. A sling was placed around the yoke and the leaf was ready for raising.

Raising the Leaves

With the standard equipment and the experience gained on previous overhauls, raising and rolling the leaves at Miraflores was largely a matter of routine. The end of each beam was blocked up during raising and lowering so that if the oil pressure failed the leaf would not be dropped more than $\frac{1}{4}$ in. One machinist and two helpers were stationed at each group of jacks while the supervisor assumed a position directly under the leaf so that each group was easily visible. The lifting was started with $\frac{3}{4}$ in. of blocking under the beams, and as the leaf was raised, the blocking was changed at a whistle signal from the supervisor. When each machinist had signaled a clearance of slightly more than $\frac{1}{4}$ in., the whistle was blown and the four groups changed the blocks together, the $\frac{1}{4}$-in. blocks being removed and a $\frac{1}{2}$-in. block put in place, making a total of 1 in. At the next whistle, the $\frac{1}{4}$-in.

block was replaced until another ½-in. block could be added. This procedure was continued until ten ½-in. blocks were in place. A solid 5-in. block was then substituted, one block at a time, for the set of ½-in. blocks. The leaf was then lowered, the pressure released, the wedges back of the jacks were removed, and the jacks reseated by hitting them lightly with a block of wood. A slight pressure was then put on the rams and the aligning wedges were replaced. The raising was continued as before until a total of 15 in. of blocking was in place, thus making a net lift of 14¼ in.

The re-alignment of the jacks was important because they are subjected to severe lateral thrusts if their axes of travel are not parallel. The average pressure required to lift the 82-ft leaves was 70 tons per jack (5 900 lb per sq in.), or a total of 840 tons, including the weight of the equipment and friction in the jacks. Fig. 7 shows the leaf partly raised and the follow-up blocking in place.

During the raising, rolling, and lowering operations, the lateral movement was checked with a transit on a line parallel to the down-stream face of the leaf. Observations were taken on targets clamped to webs of the reaction castings and perpendicular to the face. The targets consisted of 6-in. sections of a folding rule fastened to an arm welded to a C-clamp and deflections of 1/32 in. could be read easily. The unbalanced loads were controlled by proper tension in the stay cables; a signal from the transitman would indicate the direction of any lateral movement to a machinist stationed on the guide-frame who, in turn, would direct helpers, one at each turnbuckle, as to the proper tension in any cable. In this manner, the guide-frame was kept free for any emergency.

Rolling the Leaf

With the leaf raised to its full height, the weight was carried partly by the jacks and partly by the follow-up blocking. The upper and lower groups of inner H-beams and the rollers were next installed. The full weight was then taken entirely by the jacks; the follow-up blocking was removed, the outer H-beams were bolted on, the guide-bars placed over the ends of the rollers, and the beams, tying the upper tracks together, bolted in place.

To make certain that the gate would move away from the wall on the predetermined line, it was essential that the rollers be parallel and their axes normal to that line. The front and rear rollers of each train were omitted and aligning, or dummy, rollers inserted in their place. These were long enough (9 ft 8 in.) to extend through the up-stream and down-stream guide-bars of the roller train, thus insuring that the rollers in opposite trains were parallel. The axes were made normal to the line of rolling by bringing the up-stream and down-stream ends of the dummy equi-distant from the punch-marks which indicate the transverse center lines of the jacks. The dummy rollers were then withdrawn and the regular rollers inserted. Then the leaf was landed on the tracks and preparations were made for rolling it.

With the leaf resting on the roller assembly, the lifting jacks were removed, compressed, and placed near the other leaf of the pair. The pusher

jacks were hung in place (see Fig. 9), and connected to the pumping unit, and the timber shoring at the ends of the leaf was removed. Wood-blocking, consisting of 12-in. lengths of 6 by 6-in. timber, was then placed between the lower miter-end tracks and a striking point on the H-beams. These blocks were removed, one by one, as the leaf moved from the wall, thus providing a safety measure against the possibility of the leaf running away.

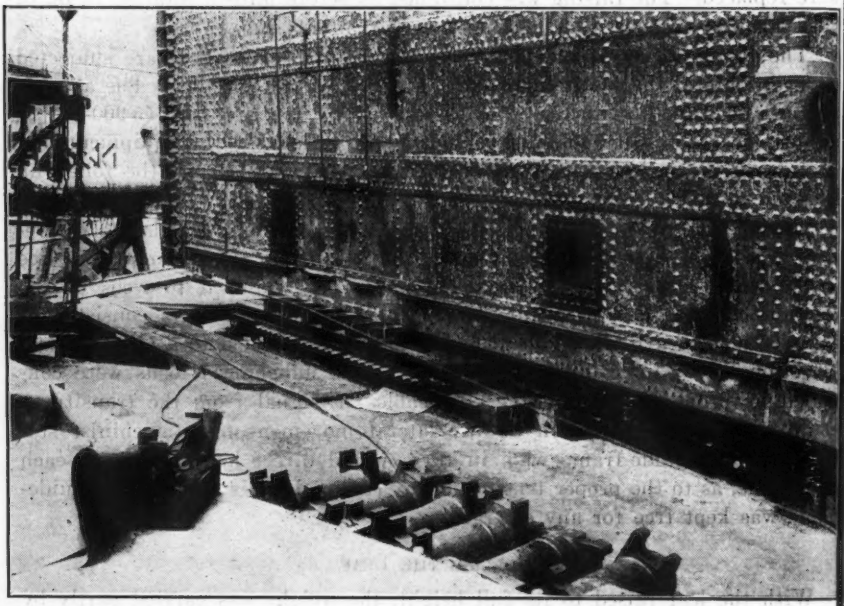

FIG. 9.—DOWN-STREAM MITER END OF LEAF RESTING ON ROLLER TRUCK.

The stay cables were carefully regulated and were arranged so that the arc of travel was divided; that is, when the leaf was 2 ft from the wall, the cables on the opposite sides of the leaf at each end were in line with each other and normal to the axis of the leaf. When the leaf had been rolled the required 4 ft, it was securely blocked in place.

The average pressure required to start a leaf was 5 tons per jack, or 10 tons in all; with the leaf rolling, the pressure dropped to $3\frac{1}{2}$ tons per jack. As an indication of the reduction in time required for raising and rolling, the following comparison is of interest:

	Raising time	Lowering time
Pedro Miguel, 1929	2 hr 55 min	37 min
Miraflores, 1933	1 hr 10 min	10 min

The leaves were rolled back and lowered in the same manner, but in reverse order. During lowering, they were landed on the follow-up blocking every $\frac{1}{2}$ in. of the last 5 in. of lowering. This was necessary because the jacks had to be reseated frequently as the leaf moved slightly away from the wall

in settling over the new pintle. Fig. 9 shows the down-stream miter end resting on the roller tracks and the pusher jacks in place for moving a leaf back to the wall.

Removal of Old Bearings

Pintle and Bushing.—The pintles and pintle bushings were removed from their sockets by the use of a cast-steel jacking ring. Two concentric rows of twelve equally spaced holes were drilled around this ring; the outer row was tapped for $1\frac{1}{4}$-in. bolts, used as jacking-screws; and the inner row of $\frac{15}{16}$-in. holes was provided for use in removing the bushing.

This ring was placed over, and welded to, the pintle; two horizontal 1-in. dowels, passed through the ring and into dowel holes in the pintle, helped to hold it in place. Turning the jack-screws against the lower pintle casting (Fig. 3) started the pintle and raised it so that it could be lifted out with the jib crane.

Although the bushings are a shrink fit in the upper casting, no difficulty was found in removing them. The following method was the most efficient: Twelve equally spaced holes were drilled and tapped into the lower edge of the bushing. Short bolts, $\frac{7}{8}$ in. in diameter, passed through the ring and set into these holes held the ring in place. The jack-screws in the outer circle bore against the upper casting (see Fig. 3). A hole was then drilled through the top of the bushing and tapped for a $1\frac{1}{4}$-in. pipe nipple. Flexible tubing was led from this fitting to a hand-operated hydraulic pump. Oil forced

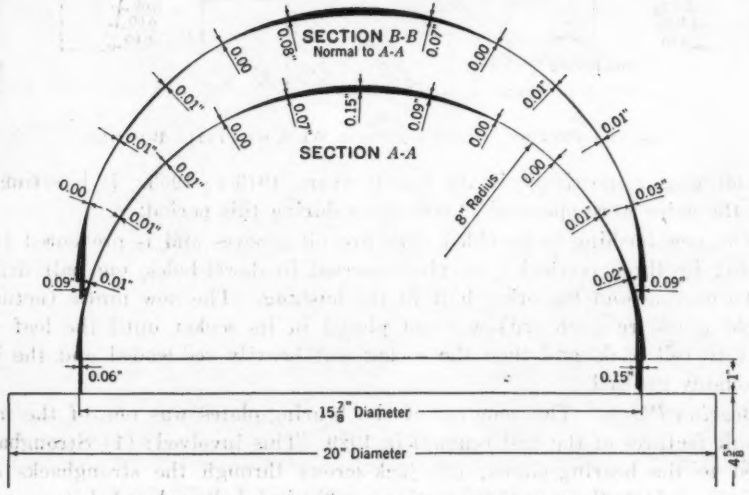

Fig. 10.—Section Showing Typical Wear on Pintle.

between the bushing and casting under a pressure of 600 to 800 lb per sq in. was usually sufficient to start the bushing. The pumping was stopped when the leakage around the bushing became excessive and the removal was completed with the ring and jack-screws.

The upper pintle castings were examined carefully for evidence of movement or shearing of bolts. The loose castings were tightened by removing the old fastenings and reaming the holes for bolts and rivets of greater diameter. The old pintles and bushings were marked with the leaf number, and lines were marked on them to indicate planes parallel and perpendicular to the lock-wall. Later, the amount of wear on each pintle and bushing was measured along these lines. Figs. 10 and 11 show typical wear on the pintle

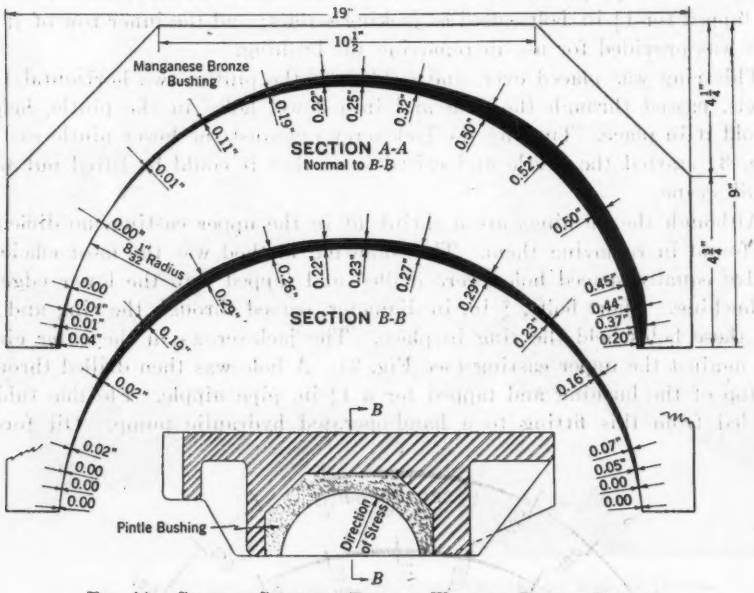

FIG. 11.—SECTION SHOWING TYPICAL WEAR ON PINTLE BUSHING.

and bushing, respectively, in the twenty years, 1913 to 1933. It is estimated that the gates were operated 71 600 times during this period.

The new bushing is provided with five oil grooves and is prevented from turning by three vertical ¾-in. pins inserted in dowel-holes, one-half drilled in the casting and the other half in the bushing. The new pintle (actually an old pintle re-machined) was not placed in its socket until the leaf was ready to roll back, and then the socket was heavily red-leaded and the ball thoroughly greased.

Bearing-Plates.—The removal of the bearing-plates was one of the most difficult features of the first renewal in 1929. This involved: (1) Strongbacks bolted to the bearing-plates; (2) jack-screws through the strongbacks and bearing against the reaction castings; (3) jack-bolts placed between the ends of the stud-bolts and the castings; and (4) hand-driven wedges to remove the plates. It was a slow and laborious method. Details of plate fastenings are shown in Fig. 2.

The bearing-plates on the gates at Miraflores were removed by a vanadium steel wedge 13 ft long, with a taper of ⅜-in. per ft, which had an offset head

keyed to a special driving head in an air-actuated pile-hammer. The wedge and hammer are shown in Fig. 12. By keying the wedge to the hammer, one crane handled both, while another crane handled the plates as they were taken off. The offset on the wedge and driving head permitted the hammer to slide down the face of the reaction castings as the wedge was driven behind the bearing-plates.

In order to start the wedge, it was first necessary to jack out the top of each bearing piece at least $\frac{1}{8}$ in. The holes in the face of the plates which were filled with babbitt metal were also drilled out. A novel means of starting the plates was tried on the last leaf repaired. A heavy pipe plug was drilled to receive a firing-pin and the end slightly counterbored to hold a 12-gauge shotgun shell. The wadding and pellets were removed and the shell was filled with $9\frac{3}{4}$ grains of smokeless powder. The plug holding the shell was screwed into the top hole in the face of each bearing-plate and the firing-pin hit with a swinging weight. The explosion was sufficient to blow the plate about $\frac{3}{4}$ in. away from the babbitt-metal backing.

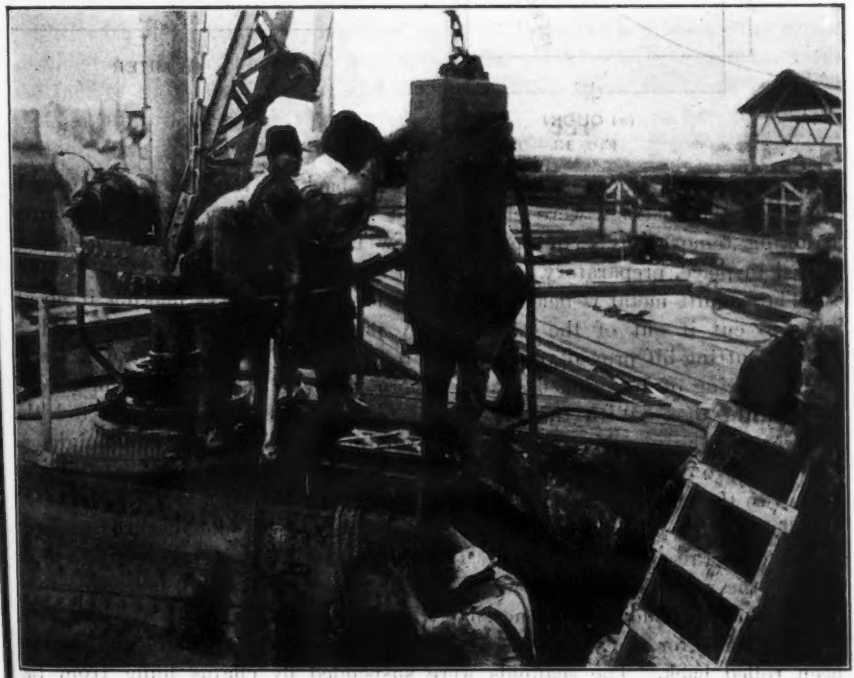

FIG. 12.—VANADIUM STEEL WEDGE AND HAMMER USED TO REMOVE BEARING-PLATES AT MIRAFLORES

The wedge and hammer were not used in removing the bearing-plates in the wall (hollow quoins) because of the possibility of loosening the reaction castings set in the concrete wall. Heavy strongbacks, made of a 4 by 4-in. bar, drilled and tapped for two capstan head jacking-bolts, were most satis-

factory for removing these plates. They were provided with two expanding-jaw clamps, centered to fit into the counterbored nut recesses of the hollow quoin-plates. Although this method was slow, the results were entirely satisfactory. Each bearing-plate was marked and the wear measured at a later date. Fig. 13 shows typical wear on the bearing-plates.

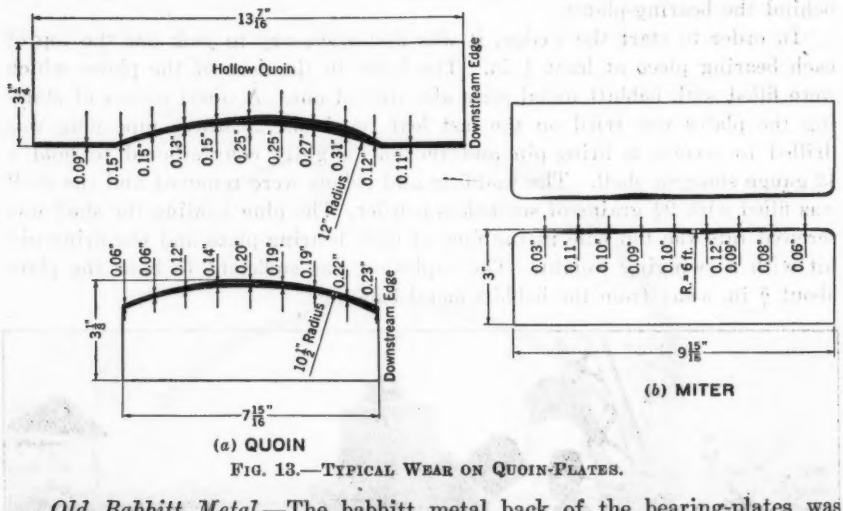

FIG. 13.—TYPICAL WEAR ON QUOIN-PLATES.

Old Babbitt Metal.—The babbitt metal back of the bearing-plates was removed, the castings were cleaned, and the burrs formed by the original caulking removed by grinding. The new bearing pieces were then loosely bolted in place, preparatory to their final setting.

The babbitt metal is held in place by dovetailed grooves making it necessary to cut it out of the castings. A pneumatic paving breaker with a special cutting-bit proved very efficient in removing it. The bit was a blacksmith forging of T-section drawn to a point and the upstanding stem ground to a knife-edge. With the bit started between the babbitt metal and the casting, it was possible to drive this cutting tool down the full height of the leaf. No one was allowed near the leaf and the babbitt metal was allowed to fall to the chamber floor as it was cut from the casting.

The bearing-plates were removed and reset from double-decked scaffolds which were raised and lowered by the jib cranes. One scaffold, shaped to fit around the reaction castings, was sufficient for the miter end, but three were required at the quoin end; one shaped to fit between the gate and the wall, and one each for the up-stream and down-stream sides when the leaves had been rolled back. The scaffolds were suspended by chains hung from the ends of an H-beam which was fastened to the top of the top reaction casting and extended over the sides of the leaf. The scaffolds could be held at any desired elevation by passing pins through chain-guides on the scaffold and the chains.

Locomotive axles, quills, and car-wheels and lengths of valve stems, which had been salvaged from obsolete equipment, were utilized in building the jib

cranes. These cranes, each with a capacity of 3 000 lb, at a radius of 13 ft, could be swung in a full circle and were equipped with a single-drum air hoist. The lower portion of one of them may be seen in Fig. 12.

Adjustment of New Bearing-Plates

The new bearing-plates (the majority of which were old plates salvaged from the Gatun repairs and re-machined) were set in essentially the same manner as in the original construction. The attempt was made to set them within 0.003 in. of their correct position, but no error greater than 0.005 in. was allowed. The plates were set in the following order: Hollow quoins, quoin-plates, and miter-plates.

Hollow Quoins.—The hollow quoins were set by gauges from a vertical piano wire. The correct position of this reference line was established with the aid of a device for centering pintles designed by engineers on the Panama Canal and consists of two parts: The cross-bar center (Fig. 14), which was set in the pintle socket; and the adjustable slide (Fig. 15), which was supported by a bent channel secured in the yoke casting. The slide was drilled with three accurately centered 0.033-in. holes, with a tensioning screw provided near each hole. One hole was at the pintle center; one, 6 in. from the face of the hollow quoins; and the other, 12 in. from the same face.

The steps in setting the device follow: (1) The cross-bar center, or "spider," was centered and clamped in the pintle socket; (2) the lower end of a No. 13 piano wire was fastened in the center of the spider and the upper end passed through the pintle center of the adjustable slide; (3) the piano wire was carefully plumbed, bottom to top, with a transit equipped with a striding level, and it was adjusted at the top by moving the slide; (4) the transit was brought to the top of the lock-wall and set up on a line normal to the chamber axis by transiting-in on the line established by the pintle center of the opposite leaf and the pintle center of the adjustable slide; (5) the center lines on the uprights at each end of the slide were brought to the transit line by revolving the slide around the center wire (this movement controlled the position of the projecting arm of the slide on which are the holes for the reference wires); (6) the reference wire was passed downward through the hole, 12 in. from the face of the plates, and fastened to the end of a projecting arm of the spider; (7) this line was plumbed, from the top to the bottom, and securely fastened (this wire is adjusted at the end of the projecting arm); and (8) as soon as the correct position of the pintle center on the movable slide had been established, reference punch-marks were placed at several points on the yoke casting. These marks were used later in bringing the yoke to its correct position.

Although the hollow quoins may be set from either the 6-in. or the 12-in. wire, the latter was found to give more accurate results. The plates are set with gauges made of sectors of stainless steel, $\frac{1}{16}$ in. thick, and have the arc of the sector cut to the radius of the hollow quoins. Scratch-marks on the gauge indicate the edges of the concave face of the plates and the position of the reference line. The gauge is held horizontally between the face of the plates and the reference wire, and the plates are turned, by temporary set

screws on the center line and the permanent stud-bolts along the edges (see Fig. 14), until the center scratch-mark coincides with the reference wire. Due to inaccuracies in the original setting of the reaction castings, it was impossible to set certain plates correctly without first planing the edges.

With the hollow quoins correctly adjusted, the babbitt metal backing was poured (as described later), the set screws were removed, the edges of the plates caulked, the nut recesses filled with concrete, and the plates thoroughly cleaned. The leaf was then rolled back and lowered. The task of keying the yoke and hinging the leaf was carefully supervised and inspected. The jacking equipment was then removed, the operating strut connected, and the leaf was free to swing.

Quoin-Plates.—The quoin-plates were set (by jacking them against the hollow quoins) with the leaf in a position 1 in. up stream from the true miter point. This setting provided a slightly closer contact between the quoin-bearings when the leaf was exactly mitered. The line of contact was centered on the hollow quoins, and the plates were then forced (with short jack-bolts between the ends of the plate studs and the reaction castings) against the hollow quoins until it was impossible to push a 0.005-in. feeler between the faces of the plates. The jack-bolts were essentially a $\frac{3}{4}$-in. hexagonal rod with one end tapped for a conical-pointed nut. The ends of the studs were countersunk to prevent the jacking nut from slipping off. In setting these quoin-plates, it was absolutely essential for the men on opposite sides of the leaf to work together, and they were provided with portable telephones for easy communication. To set the quoin-plates in the exceedingly cramped working space required some of the most arduous work of the entire over-haul. In order to pour the babbitt metal behind the plates, it was necessary to swing the leaf to the full open (against the wall) position.

Miter-Plates.—The miter-plates were set early in the morning, as follows: (1) Both leaves of the pair were brought to the miter point; (2) the top and bottom plates were set so that the thickness of the babbitt metal was the same on each; (3) the leaves were opened partly and a reference wire stretched from top to bottom and 2 in. from the face of the plates on one leaf; (4) the plates were set by a 2-in. gauge from the reference wire, and the babbitt metal was poured; (5) at the same hour of a following morning, the leaves were mitered again; (6) the bottom plates of the second leaf were brought in contact with those of the first leaf and the top of the top plates was separated by a $\frac{1}{32}$-in. filler; and (7) the plates were set and babbitted. The effect of this setting was to form a V-shaped opening, $\frac{1}{32}$ in. wide at the top, which served to compensate slightly for the effects of temperature expansion and wear.

Pouring Babbitt Metal.—After each set of bearing-plates had been adjusted, the spaces behind the plates were filled with babbitt metal; this work of pouring the molten metal from the small and swinging scaffolds was the most dangerous operation of the over-haul.

The babbitt metal was heated in a double-ended, oil-burning, brick-lined, steel box furnace, placed on the chamber floor. Two openings were provided

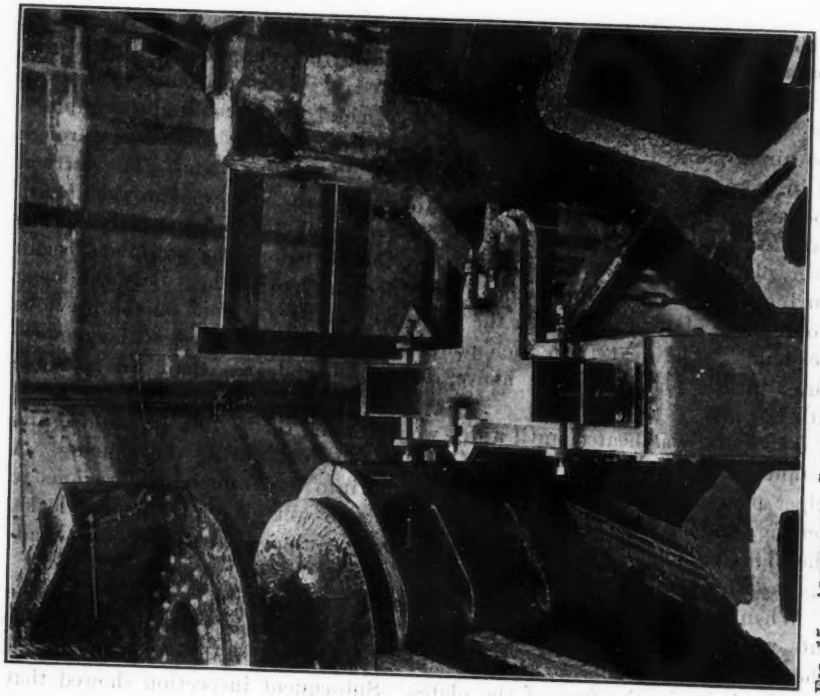

FIG. 15.—ADJUSTABLE SLIDE FOR PINTLE CENTERING DEVICE, SUPPORTED BY A BENT CHANNEL SECURED IN A YOKE CASTING.

FIG. 14.—CROSS-BAR OF PINTLE CENTERING DEVICE SET IN PINTLE SOCKET.

in the top for the pots, the compartment under each pot being fired with separate air-oil burners, with the smoke flue in the center of the furnace.

The spaces behind the plates were heated by compressed air to dry out any moisture and to prevent the molten metal from cooling too rapidly. The air circulated through the coils in the flue and was conducted through a 1-in. pipe to another vertical pipe near the edge of the plates. The vertical pipe extended about 25 ft above the bottom plate and was provided with branches leading to tapped holes in the face of the bearing-plates. The lower plates were heated for about two hours before the workmen began pouring. Experience showed that it was necessary to pre-heat only the lower two or three plates rather than the entire height, as was done on the previous over-hauls. The heat from each pour was sufficient to increase the temperature, in the plates directly above, to a higher degree than was possible with hot air.

The metal was heated until a dry wooden stick thrust into it would ignite instantly. Four men were required on the scaffold during the pouring. A helper poured a small quantity of kerosene into the funnel just previous to pouring, and he was provided with a supply of wet clay for stopping leaks. The signalman and the crane operator were equipped with telephone headsets in order to minimize the danger of a misunderstood hand signal.

When the plates had cooled, the asbestos caulking was removed, and the nuts on the studs were tightened to take up the shrinkage of the babbitt metal. The edges of the plates were then caulked with lead, and lead plugs driven into the holes in the face of the plates. Subsequent inspection showed that the plates had been set in so accurately that no grinding of the faces was necessary.

Changes to Original Gate Design

Automatic Pintle-Oiling Device.—In the original design no provision was made for oiling the pintle. The new bushings are provided with five oil grooves leading to an oil passage drilled through the bushing and upper pintle casting. An extra heavy 1-in. brass tube leads from this oil passage to an oil pump on top of the leaf, which is placed so that the plunger is actuated by the leaf-operating strut when the leaf approaches the miter position.

It is a question whether the life of the bearing-plates is increased by oiling the pintle. The bushing must yield with the wear on the quoin-plates in order that the leaves may close to secure the necessary arch effect when there is a head of water behind them. However, with oil on the pintles the leaves swing more smoothly.

Concrete Sills.—The fixed wooden sills (Fig. 5) were replaced with concrete sills because the wooden ones were subject to the attacks of the teredo and require expensive replacement. The wooden clapping sills on the gate were increased in thickness to allow for the additional trimming. Two rubber pads, $1\frac{1}{2}$ in. thick, were set 1 in. into the face of the clapping sill near the miter end to prevent the leaves from slamming too forcibly against the concrete sill.

Sealing Bead on Bearing-Plates.—To secure a truly water-tight bearing, projecting beads of babbitt metal were poured into small dovetailed grooves in the faces of the miter and quoin bearing-plates. This was the last operation before the water was let into the chamber. At the miter end, two grooves were located 2 in. each side of the center line of the plates on one leaf only. At the quoin end, one groove was placed $2\frac{3}{8}$ in. down stream from the center of the quoin-plates on each leaf.

General Observations

This paper describes the various operations in reference to one leaf only. It should be understood, however, that three complete sets of equipment were provided, and that the bearings were renewed simultaneously on six leaves in each chamber. The work was done continuously by three 8-hr shifts, six days per week. Numerous flood-lights provided ample illumination for all night work with the exception of raising or lowering the leaves, which was done only in daylight.

Each group of leaves was in direct charge of a foreman, with a general supervisor responsible for the three groups. Engineers and inspectors on each shift were responsible for the correct layout, proper use of the equipment, and the final adjustment of the new bearings.

The following non-related comments are worthy of attention: (*a*) Extensometer readings, taken during the gate-raising, gave results that checked closely with the computed stresses; (*b*) the average cost per leaf (direct charges) was $25 693.55, including labor, material, and special equipment; and (*c*) at the conclusion of the Miraflores over-haul, the equipment was repainted, re-marked, and shipped to Gatun for storage until the next over-haul on those locks.

The equipment and methods for renewing the bearings on the Panama Canal gates have become practically standardized, and only very minor changes in detail are probable.

Acknowledgment

The plant and equipment for overhauling the gates were originally designed by E. S. Randolph, M. Am. Soc. C. E., and many of the improvements are due to his recommendations, made at the conclusion of the trial raising at Pedro Miguel. The unit stresses, as given in this paper, and other supporting data, are taken from Mr. Randolph's report to the Assistant Engineer of Maintenance of The Panama Canal, in 1929. The writer, as Junior Engineer, was in charge of engineering and inspection on one of the eight-hour shifts.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS

By Darwin Wadsworth Townsend, M. Am. Soc. C. E.[1]

Synopsis

This paper involves the consideration of no new hydraulic theory or formulas. The problem implied in the title includes an analysis of flow conditions that differ from those encountered in the usual channel problems, because the introduction of diffused air tends to retard velocity.

The presentation is semi-hypothetical, and the various functional values expressed are, strictly speaking, relative only. As such they are susceptible to direct application and use on the basis of any value of n that may be selected. The assumption is made that a roughness coefficient of $n = 0.113$, in Kutter's formulas, is reasonably applicable for all practical purposes to channels constructed in a manner similar to that described herein, and for unaerated sewage.

The purpose of the studies was to determine the extent to which flow is retarded when diffused compressed air is introduced into a flowing mixture of sewage and activated sludge, the total suspended matters in the mixture varying inappreciably from a normal value of 3 000 ppm. Numerous previous experiments have led the writer to conclude that this solids content does not change the hydraulic properties of the mixture from those of water alone.

The tests indicate definitely that: (*a*) Resistance to flow increases because of diffused air; (*b*) as the velocity of flow decreases, the resistance increases; and (*c*) the use of higher values of n is necessary because of increased resistance. The subject has been treated sufficiently herein to enable readers to comprehend the nature of the problem and the applicability of the results to problems of design.

Sanitary engineers are familiar with the fact that the activated sludge process of sewage treatment involves primarily the injection and diffusion of relatively large volumes of compressed air into the liquid sewage under-

Note.—Discussion on this paper will be closed in **April, 1934**, *Proceedings*.

[1] Acting Chf. Engr., Sewerage Comm., Milwaukee, Wis.

going treatment, and that the countless ascending streams of tiny air bubbles injected at the tank or channel bottom offer resistance to the forward movement of the flowing sewage. This latter fact is obvious to one who has observed the surface appearance of an aeration tank or an aerated channel in which may be seen innumerable and continuously changing cross-currents, counter-currents, and swirls—all of which are indicative of resistance to the flowing stream.

It was because of this condition and the realization of its significance in hydraulic design, complicated by this flow-disturbance factor, that the writer undertook to determine the extent to which flow was retarded when diffused compressed air was introduced in the feed channels of the existing sewage treatment plant at Milwaukee, Wis. It is worthy of note that the selection of $n = 0.020$ in Kutter's formulas, was considered ultra-conservative in 1928 when the design of the present plant was begun, and that subsequent operation proved conclusively that this coefficient was inadequate.

The intent of this paper is not to formulate basic laws applicable to hydraulic properties of water or sewage, mechanically impregnated with diffused air. As a matter of interest, however, it may be assumed that: (1) Water impregnated or saturated with air has a specific gravity less than water alone; (2) because of this probable fact, it possesses hydraulic properties differing from those of plain water; and (3) if Assumptions (1) and (2) are correct, the hydraulic laws governing the flow of water would need to be modified somewhat, in order to classify this particular medium properly.

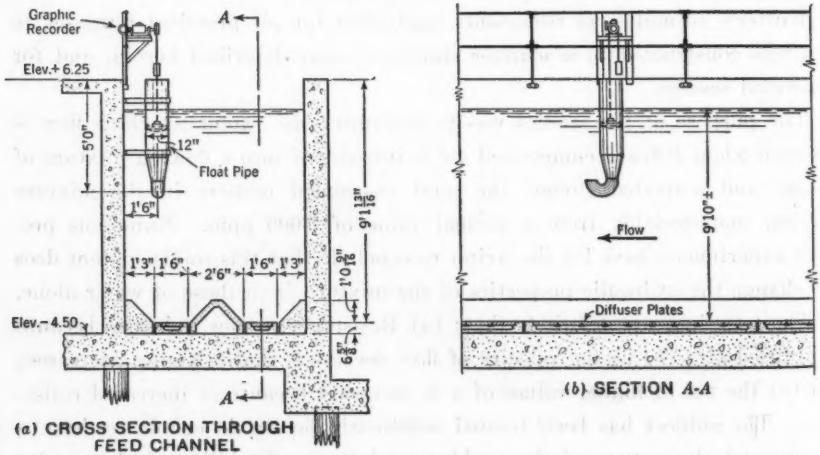

FIG. 1.—GRAPHIC RECORDER IN FEED CHANNEL.

However, in this paper, the subject must be treated from the viewpoint of an increased roughness coefficient, assuming the flowing medium to possess the hydraulic properties of water alone.

For the purpose of determining the roughness coefficient applicable to use in Kutter's formulas, two head-level recording stations were established, 123.34 ft apart, in a section of an aerated-sewage feed channel where

the cross-section was uniform. In this section, and under the conditions of the test, the cross-section area was 75.39 sq ft, the wetted perimeter was 27.47 ft, and the mean hydraulic radius was 2.774 ft. The floats for operating the recording instruments were mounted in 12-in. pipes with upturned entrances,

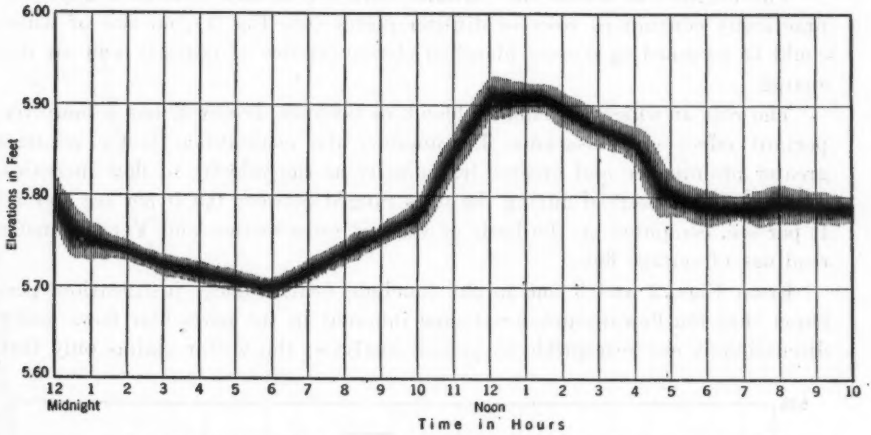

FIG. 2.—HEAD LEVEL CHART IN FEED CHANNEL, STATION NO. 1, JULY 8, 1928.

6 in. in diameter, at the bottom, to exclude the ascending streams of air bubbles, as much as possible.

The type of float-chamber finally adopted (see Fig. 1) was one of several under consideration. The representative reproduction shown in Fig. 2, of one of the many graphic records taken at the up-stream recording station,

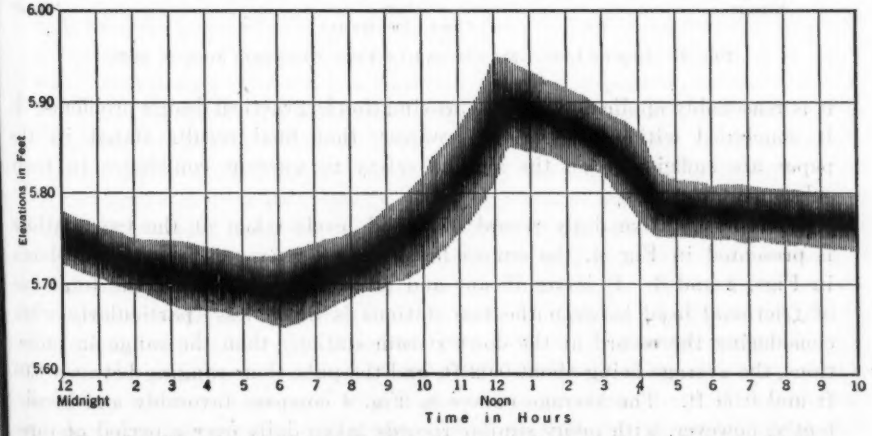

FIG. 3.—HEAD LEVEL CHART IN FEED CHANNEL, STATION NO. 2, JULY 8, 1928.

demonstrates the degree of quiescence. It will be noted that the pulsations in the float compartment were practically continuous and that they varied in vertical movement between 0.02 ft and 0.05 ft. The records at the down-stream station, a graphic representation of which is shown in Fig. 3, indicated

pulsations of greater magnitude than those at the up-stream station because of an obviously greater degree of diffused air disturbance at this point. The vertical movement of these continuous pulsations may be seen to vary between 0.06 ft and 0.09 ft.

The channel in which the stations were established contained a pair of practically continuous rows of diffuser plates (see Fig. 1), no two of which could be assumed to possess identical characteristics of porosity and air discharge.

The rate at which air bubbles blend, or coalesce, however, has a more important effect on pulsations. Presumably, the coalescence factor assumes greater prominence and greater irregularity as the velocity of flow increases. The velocities observed during the tests ranged between 0.9 ft per sec and 1.3 ft per sec, computed on the basis of the wet cross-section and Venturi meter readings of sewage flow.

From Figs. 2 and 3 one might conclude (with logical justification, perhaps) that the flow-disturbance factor inherent in the particular flows under discussion is not susceptible to precise analysis; the writer claims only that

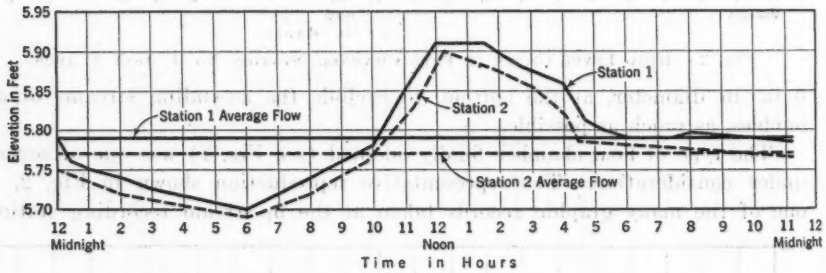

FIG. 4.—LIQUOR-LEVEL ELEVATIONS IN FEED CHANNEL, JULY 8, 1928.

it is reasonably applicable as an approximation in practical design problems. It is concluded without hesitancy, however, that final results stated in the paper are sufficiently on the side of safety to warrant confidence in their subsequent use.

A representative daily record of liquid levels taken at the two stations is presented in Fig. 4, the curves being values averaged from those shown in Figs. 2 and 3. It is significant and unusual, perhaps, that the total loss of frictional head between the two stations is much less (particularly when considering the record at the down-stream station) than the range in pulsations, the average being about 0.02 ft, and the pulsations ranging between 0.06 ft and 0.09 ft. The average curves in Fig. 4 compare favorably and consistently, however, with many similar records taken daily over a period of three weeks at the two stations. The average friction loss between these stations varied only slightly under comparable flows, generally about 0.02 ft to 0.03 ft.

A fact established quite definitely in connection with the loss of head determinations was that the coefficient of roughness (n-values) increased as

the velocity of flow decreased. The approximate trend is indicated by the upper part of Curve 1, Fig. 5, which was constructed from liquid-level observations made in July, 1928, on the ridge-and-furrow aeration system. The value of n is 0.035 at 1.3 ft per sec and 0.0425 at 0.9 ft per sec, which

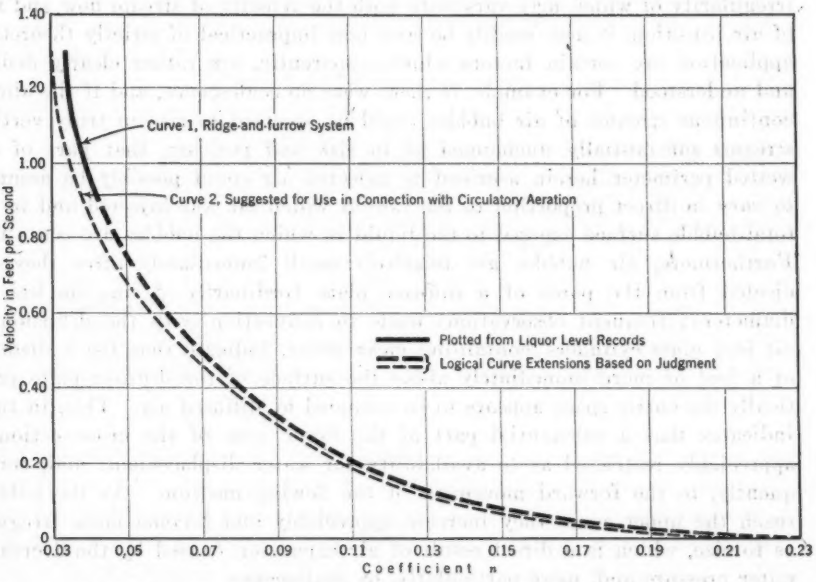

FIG. 5.—RELATIONSHIP BETWEEN VALUE OF n AND FLOW VELOCITIES IN AERATED FEED CHANNEL (WITH ESTIMATED EXTENSION BELOW VELOCITY OF 0.90 FOOT PER SECOND).

indicates an increase in n of about 25% for a reduction in velocity of about 30%; and, *vice versa*, a decrease in n of 18% for an increase in velocity of 44%, considering only the two terminal points of the curve, as shown.

A hypothetical cause-and-effect theory may be ascribed to this varying roughness coefficient which, from the standpoint of practical merit and application, may possibly possess little to warrant any elaborate exploitation. Certain factors of influence appear to be sufficiently well defined, however, at least to warrant mention. Upon analysis, it becomes apparent that the hydraulic properties of a channel cross-section may be assumed to be changed artificially through the continuous presence in the flowing medium of the countless ascending streams of air bubbles, the degree of change depending primarily on the rate of diffused air injection. This statement is made with particular reference to an artificially and obviously decreased cross-section and the resultant increased wetted perimeter, ascribing to an air-bubble surface the same, or nearly the same, value of unit wetted perimeter as that which applies to the wetted surface of the channel structure itself. On the basis of this hypothesis, it is obvious that the "air"-wetted perimeter would be increased theoretically in proportion to the rate of diffused air injection, as previously noted, and, as a consequence, hydraulic radii would decrease in

somewhat the same proportion, ignoring for the time being the extent of the fixed wetted surface of the channel structure and its definitely known influence upon the hydraulic properties of the channel.

Because of the tendency of the air-bubbles to coalesce the degree and irregularity of which may vary with both the velocity of stream flow and rate of air injection, it may readily be seen how impractical of strictly theoretical application are certain factors which, apparently, are rather clearly defined and understood. For example, if there were no coalescence, and if the almost continuous streams of air bubbles could be assumed to rise in truly vertical streams substantially unchanged as to size and position, that part of the wetted perimeter herein ascribed to injected air could possibly be assumed to vary in direct proportion to the rate at which air was injected and to the total bubble surface exposed to the liquid in which the bubbles are contained. Furthermore, air bubbles are relatively small immediately after they are ejected from the pores of a diffuser plate (ordinarily $\frac{1}{16}$ in., or less, in diameter); frequent observations made in connection with the diffusion of air into glass cylinders containing clear water, indicate that for a distance of a foot or more immediately above the surface of the diffuser plate practically the entire space appears to be occupied by diffused air. This, in turn, indicates that a substantial part of the lower zone of the cross-section is appreciably restricted as to availability for water displacement, and, consequently, to the forward movement of the flowing medium. As the bubbles reach the upper zones they increase appreciably and become more irregular as to size, which is a direct result of air expansion caused by the decreased water pressure and, more particularly, by coalescence.

During the period in which liquid-level records were being obtained, air was forced into the flowing admixture of sewage and activated sludge at a constant rate of 1 cu ft of free air per diffuser plate per min, and upon a strictly assumed theoretical basis the air-bubble resistance to flow was constant. However, inasmuch as resistance to flow, or the roughness coefficient, has been shown to increase as the velocity of flow decreased, it follows that a decrease in roughness coefficient with a fixed rate of air injection, may be attributable to an increased degree of coalescence as a direct result of increased velocity.

Table 1 has been compiled in order to show more clearly the possible extent to which the hydraulic properties of a channel are subject to assumed modification due to the air-injection feature. The values are based on: (*a*) The velocities that obtained during the period in which records were taken (0.9 ft to 1.3 ft per sec); and (*b*) the friction head per linear foot of channel, using $n = 0.013$ as a basis (see Table 1, Columns (1) and (2)).

Values of n and C, shown in Columns (3) and (4), were computed. Column (5) contains relative velocity reductions as a result of increased n-values over and above $n = 0.013$; in other words, the velocity values shown in this column are those which. if used in connection with values in Column (3), would produce the same friction heads as those shown in Column (2), and are translations of values of Column (3) into terms of relative velocity.

Furthermore, translations on the foregoing basis for relative mean radii, *MR*, and wetted perimeters, are shown in Table 1, Columns (6) and (7), respectively. These data indicate the degree to which the two hydraulic

TABLE 1.—RELATIVE VALUES OF AERATED CHANNEL PROPERTIES, USING AS A BASIS AN UNAERATED CHANNEL OF CONSTANT CROSS-SECTION AND THE VALUE OF $n = 0.013$

(Slide-Rule Computations)

Velocity, in feet per second (1)	Friction head per foot for $n = 0.013$ (2)	Value of n of curves shown on Fig. 5 (3)	Value of C for n of curves shown on Fig. 5 (4)	Velocity for friction head corresponding to n on curve (5)	Value of mean radii corresponding to the velocity at $n = 0.013$ (6)	Wet perimeter corresponding to mean radii at $n = 0.013$ (7)	Percentage decrease of mean radii at $n = 0.013$ (8)	Percentage increase of wet perimeter at $n = 0.013$ (9)	Percentage increase of velocity at $n = 0.013$ (10)
1.40	0.0000392	.....	.....	.....	.....	.....	.....	.....	.....
1.30	0.0000338	0.0348	49.8	0.481	0.66	114.2	416	416	270
1.20	0.0000288	0.0351	49.5	0.442	0.65	116.0	423	423	272
1.10	0.0000242	0.0363	48.6	0.396	0.63	119.5	436	436	278
1.00	0.0000200	0.0388	47.0	0.350	0.61	123.5	451	451	286
0.90	0.0000162	0.0424	44.6	0.298	0.565	133.5	486	486	302
0.80	0.0000128	.....	.....	.....	.....	.....	.....	.....	.....

functions may be assumed to be altered, on the hypothesis that the wetted perimeter may be increased and the hydraulic radius decreased, due to the presence of vertically rising streams of air-bubbles which are assumed to offer flow resistance comparable to that of a surface with a fixed wetted perimeter. Column (8) and (9) (Table 1) indicate the same as Columns (6) and (7), but are expressed in terms of percentage decrease and percentage increase in the functional values noted.

In Column (10) (Table 1), is noted in percentage the extent to which the velocities would be accelerated if the resistance causing the increased n-values were removed, and $n = 0.013$ were substituted.

Much more of possible interest might be presented relative to: (1) The cause and effect of the air injection; (2) the resulting retarding current disturbance factor; and (3) its influence on hydraulic problems of design. Nevertheless, it is hoped that this discussion has been in sufficient detail to solicit appreciation of, and to direct attention toward, the significance attributable to the flow characteristics of sewage impregnated with air.

In the system of aeration using circulatory flow, relatively few diffuser plates are needed as compared with the so-called ridge-and-furrow system in use at the plant in Milwaukee. It is the writer's judgment that less retardation of flow might be anticipated in connection with that system. In Fig. 5, Curve 2 has been plotted arbitrarily and is suggested for use as noted in connection with circulatory aeration.

The reasons for anticipating lower n-values are as follows (and it is to be clearly understood that these reasons are formulated and offered in the absence of supporting data other than that of attempted logical analysis): In visualizing the movements of sewage admixed with activated sludge under-

going aeration where diffuser plates are numerous, as in the ridge-and-furrow system, there is in the general proximity of each diffuser plate a complete induced cycle of ascending and descending currents, the total length of ascent and descent traversed being approximately equivalent to twice the depth of the overlying liquid. Obviously, then, the more numerous the diffuser plates are the more numerous will be the current cycles, the greater will be the total distance traversed in a given time, and the greater will be the velocity of travel.

As a result of the higher and more frequent upward and downward current velocities, greater resistance to forward or horizontal flow might reasonably, and obviously, be expected. Otherwise stated, the angle of the resultant of the horizontal and vertical flow components referred to the horizontal might be expected to increase appreciably as the vertical velocities increase, the velocity of horizontal flow remaining constant.

In the strictly circulatory or spiral aeration system whether in a tank or sizable channel, fewer diffuser plates are used and, consequently, fewer ascending and descending flow-retarding currents are present. In this system in which the diffuser plates are located along one side of a tank or channel, a circulatory flow around the entire periphery is induced as a result of the injection of air. This creates a sufficient head at the liquid surface over the diffusers to set up the circulatory flow from which the system derives its name.

Little or no well-defined current other than a relatively slow forward movement is present in the "core" of the liquid, or in that part somewhat removed from the well-defined peripheral currents. Therefore, in the absence of currents in these zones (which, in area, comprise the major part of the total cross-section) there is little or no resistance to forward flow, the lateral restricting currents in the forward flow existing in the general proximity of the sides and bottom of the tank and at the liquid surface.

The range of channel velocity herein noted (0.9 ft to 1.3 ft per sec), is that in which activated sludge particles would ordinarily be precipitated to the bottom of a channel, in a liquid sewage medium. The diffusion of air into the mass obviously is made necessary when the range of flow is great and when there is insufficient head to produce higher and self-cleansing velocities.

Thus far, the paper has treated mostly upon flow conditions in supply channels, on the basis of the records mentioned. In aeration tanks where exceedingly low velocities occur (little in excess of 0.02 ft to 0.03 ft per sec) a curve showing n-values plotted against velocity would be expected to approach the horizontal closely as the velocity nears zero. To investigate this phase a base rate of curvature was established from readings between $v = 0.90$ and $v = 1.30$ ft per sec, and from this Curve 1 was extended toward values of low velocities. This projection (Fig. 5) showed as an asymptote to the horizontal zero axis; for example, a value of $n = 0.20$ was found to correspond with a velocity of only 0.02 ft per sec. This chance projection confirming logical expectations is of considerable interest.

The projection of this curve is of considerable interest, since besides showing working n-values at various velocities its horizontal tangential point, falling in the immediate vicinity of zero velocity as it does, lends additional confidence to the manner and accuracy with which the liquid-level records were taken and the subsequent computations made. A lower curve has also been constructed for use in connection with circulatory aeration flows.

In closing, the writer wishes to acknowledge with thanks the assistance of William Landsiedel, Assoc. M. Am. Soc. C. E., who, under the writer's direction, prepared and arranged the data in the various diagrams.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

WILLIOT EQUATIONS FOR STATICALLY INDETERMINATE STRUCTURES

IN COMBINATION WITH MOMENT EQUATIONS IN TERMS OF ANGULAR DISPLACEMENTS

BY CHARLES A. ELLIS,[1] M. AM. SOC. C. E.

SYNOPSIS

If the Williot diagram served only as the graphic solution for the deflections of a truss, it would still hold a prominent place among the analytic methods used by the designing engineer; but its usefulness is by no means so circumscribed. When combined with Maxwell's theorem of reciprocal displacements it is the quickest solution for statically indeterminate reactions in continuous spans, two-hinged arches, and similar structures. It also serves in determining the angular displacements of members in secondary stress analysis.

Another inherent quality in the Williot diagram recently discovered should be of considerable assistance in stress analysis. So far as known, it is a new idea in the theory of structures. Like many other inventions, necessity was the mother of this one. In studying a large and unusual structure, the writer encountered a problem in stress distribution that would not yield to an exact or even to a satisfactory solution because of the lack of sufficient equations, until he discovered them in the geometric properties of the Williot diagram.

This paper recites the circumstances leading up to this dilemma, describes the development of the Williot strain equations (Equations (13) to (21), inclusive, and Equations (29), (30), (36), and (37), and shows how they were combined with moment equations expressed in terms of angular displacements to meet the situation.

NOTE.—Discussion on this paper will be closed in April, 1934, *Proceedings*.

[1] Evanston, Ill.

Certain abnormal structures, such as some types of towers for suspension bridges, do not lend themselves readily to solution for statically indeterminate stresses. Special methods are found necessary. This paper describes the expedient of expressing moments in terms of angular displacements as the vehicle by which such problems may be carried to a solution. The expressions for moments at the ends of the member, *IK* (Fig. 1), for example, are usually written in the form:

$$\left.\begin{aligned} M_{ik} &= \frac{2EI}{l_{ik}}(2\phi_i + \phi_k - 3\psi_{ik}) \\ M_{ki} &= \frac{2EI}{l_{ik}}(2\phi_k + \phi_i - 3\psi_{ik}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, E is the modulus of elasticity, and I represents the constant moment of inertia. When I is not constant throughout but has the magnitude, I_{ik}, for the upper half and I_{ki} for the lower half, and if $I_{ki} = nI_{ik}$, the expressions may be written, as follows:

$$\left.\begin{aligned} M_{ik} &= \frac{8nEI_{ik}}{l_{ik}(n^2+14n+1)}[(n+7)\phi_i + (2n+2)\phi_k - (3n+9)\psi_{ik}] \\ M_{ki} &= \frac{8nEI_{ik}}{l_{ik}(n^2+14n+1)}[(7n+1)\phi_k + (2n+2)\phi_i - (9n+3)\psi_{ik}] \end{aligned}\right\} \quad \ldots\ldots (2)$$

Whatever the variations in I, the moments may still be expressed as functions of the angles, ϕ and ψ. Thus,

$$\left.\begin{aligned} M_{ik} &= a\phi_i + b\phi_k - c\psi_{ik} \\ M_{ki} &= d\phi_k + e\phi_i - f\psi_{ik} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

the coefficients, a to f, being expressed numerically when the variations in I are known.

Since the writing of these equations requires known moments of inertia, the application of this method postulates a designed structure. If the structure is statically indeterminate, in order to compute the approximate stresses required for designing, it is necessary to formulate a tentative design by making certain assumptions as to the probable behavior of the structure, which will render it statically determinate. This tentative design will contain all the errors made in the assumptions. It is desirable, therefore, that the assumptions closely approximate the actual conditions. This is not a difficult matter for the experienced engineer when dealing with normal structures.

Other structures, however, may be so novel or peculiar in type or so extraordinary in size, or both, that they do not come in this category. The writer found this to be true when designing the towers for a suspension bridge, given to him in the form shown in Figs. 2 and 3. This pattern of wind bracing makes the structure many times statically indeterminate. It may not be a good example of economic or efficient bracing, but it illustrates the discovery made while designing it.

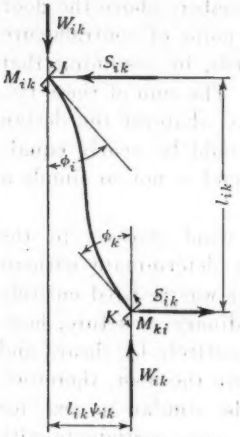

Fig. 1

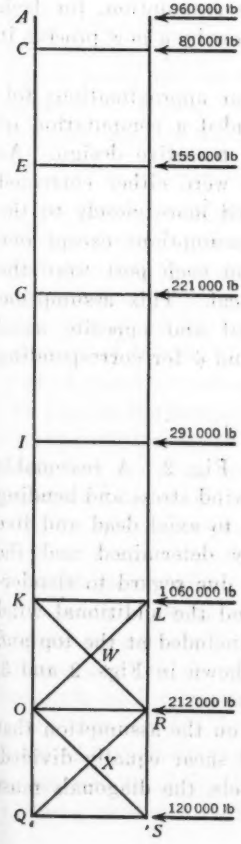

FIG. 2.

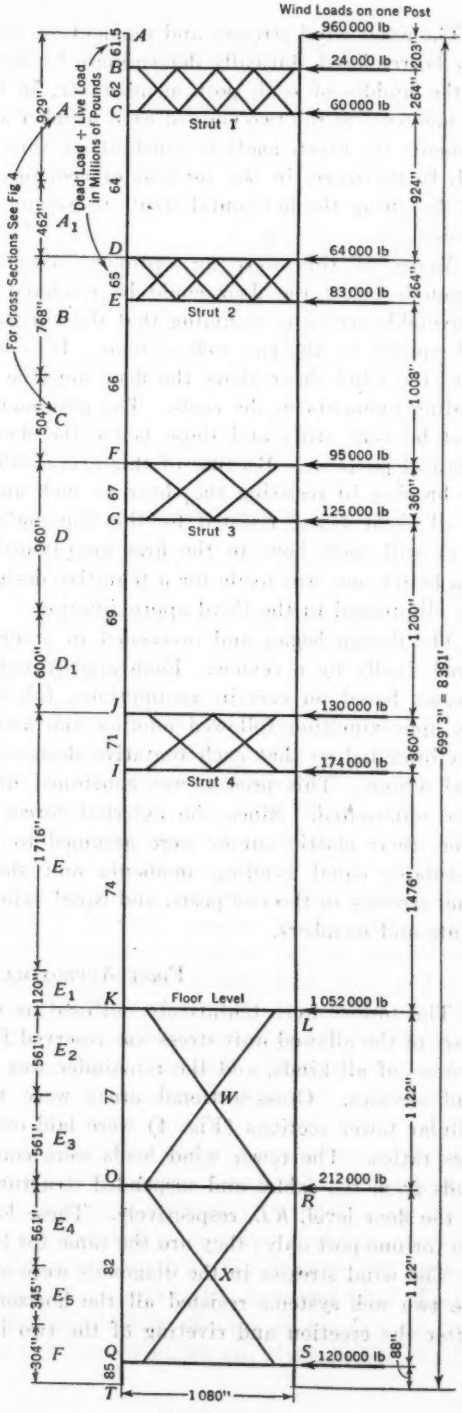

FIG. 3.

The axial wind stresses and moments in the post members above the floor may be rendered statically determinate by assuming a point of contraflexure at the middle of each post member; or, in other words, by assuming that the moments at the two ends of each member are equal. The sum of these two moments for given loads is constant, or very nearly so, whatever the design and, furthermore, in the interest of economy, they should be nearly equal; but designing the horizontal struts to accomplish this end is not so simple a problem.

Again, if this were an ordinary structure, the wind stresses in the diagonals below the floor could be rendered statically determinate without appreciable error, by assuming that the horizontal shear was resisted entirely and equally by the two web systems. It is not an ordinary structure, however; the wind shear above the floor must be resisted entirely by shears and bending moments in the posts. The post members above the floor, therefore, must be very stiff; and those below the floor must be similar in size for practical purposes. Because of this extra stiffness, the posts participate with the bracing in resisting the shear to such an extent that the assumption of of all shear being resisted by the diagonals becomes a gross error. This paper will show how, in the first approximation, this assumption, for lack of a better one, was made for a tentative design, and how by a new process it was eliminated in the third approximation.

The design began and proceeded in a series of four approximations followed finally by a review. Each approximation included a computation of stresses based on certain assumptions, followed by a tentative design. As one approximation followed another the assumptions were either corrected or eliminated so that each tentative design approached more closely to the final design. This process was continued until all assumptions except one were eliminated. Since the external forces acting on each post were the same, their elastic curves were assumed to be identical. This assumption postulates equal bending moments and shears, equal and opposite axial wind stresses in the two posts, and equal values of ϕ and ψ for corresponding joints and members.

First Approximation

The towers were tentatively outlined as shown in Fig. 2. A reasonable share of the allowed unit stress was reserved for axial wind stress and bending stresses of all kinds, and the remainder was allocated to axial dead and live load stresses. Cross-sectional areas were tentatively determined and the cellular tower sections (Fig. 4) were laid out, giving due regard to slenderness ratios. The tower wind loads were computed and the additional wind loads from the cables and suspended structures were included at the top and at the floor level, *KL*, respectively. These loads, as shown in Figs. 2 and 3, are for one post only; they are the same for both posts.

The wind stresses in the diagonals were computed on the assumption that the two web systems resisted all the horizontal wind shear equally divided. After the erection and riveting of the two lower panels, the diagonals must

participate with the posts in supporting whatever vertical dead and live loads are added to the posts thereafter.

Let p = the dead and live load unit stress in the post; p', the participation unit stress in the diagonal; and θ, the angle, OKW (Fig. 2). Then, $p' = p \cos^2 \theta$. This dead and live load participation stress was estimated, thus

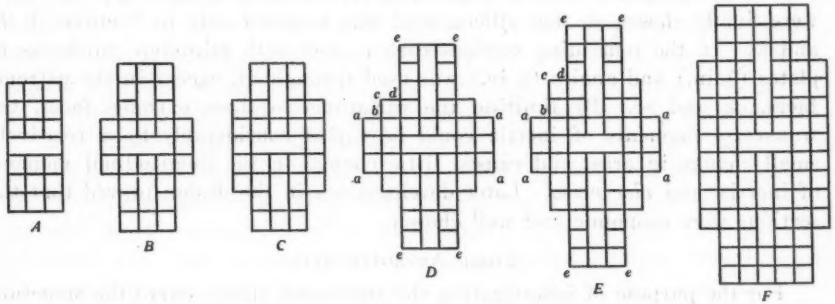

FIG. 4.—TYPICAL CROSS-SECTIONS OF TOWER (SEE FIG. 3 FOR LOCATIONS).

determining the allowable unit wind stress, and the diagonals were designed accordingly. The member, KL, was designed to meet the requirements of a floor-beam, and the two lower horizontal members were designed as nominal members free from primary stress. Except for the horizontal struts above the floor the first tentative design was complete.

SECOND APPROXIMATION

In the next computation, the transverse wind stresses and moments in the posts were rendered statically determinate by assuming a point of contraflexure at the mid-height of each panel. Analysis of the cables and suspended structure showed that the tower must submit to a longitudinal deflection at the top equivalent to 17 in. in either direction. The longitudinal moment of inertia of each post member was computed. The section of the post changed at each panel point. A parabola tangent to the center line at the base of the post and having an abscissa of 17 in. at the top, was assumed for the elastic curve. The moment of the eccentric dead, live, and wind loads was computed at each panel point and combined with the moment of a horizontal force, H, acting at the top. An expression for the deflection at the top was developed by the area-moment method and equated to 17 in. With this equation solved for H, the total longitudinal moment was computed for each panel point.

The axial dead, live, and wind stresses and the transverse and longitudinal moments were now approximately known. The resulting unit stresses at cross-sections above and below each panel point were computed for critical points in each cross-section, and the first tentative design was thus revised.

In this revision further attention was given to the number and arrangement of cells in the various cross-sections, and to the area and kind of metal to be used in each. These studies resulted in the use of six typical cross-sections, as shown in Fig. 4, the locations of which are given in Fig. 3. The

saddle casting governed the size of Section A, and the minimum allowed radius of gyration for the lower half of the post determined the size and shape of Sections E to E_5, which are alike, except for the thickness of metal in the faces, *aa* and *ee* (Fig. 4). A minimum allowed thickness of metal fixed the minimum area of metal in any given section, and this fact greatly influenced the grade of steel used. The intermediate Sections B, C, and D, were finally chosen so that silicon steel was required only in Sections A, B, and C. In the remaining sections carbon steel with minimum thickness of plates ($\frac{7}{8}$ in.) and angles ($\frac{5}{8}$ in.) was used throughout, except in the extreme faces, *aa* and *ee*. By limiting the variations to these extreme faces, the transverse moments of inertia could be varied considerably by a relatively small change in area, and caused little change in the longitudinal moment of inertia, and *vice versa*. Later developments in the design proved that the sections were economic and well chosen.

Third Approximation

For the purpose of investigating the transverse elastic curve the structure was divided into two parts at a point just above the member, KL. The wind strains were computed from stresses as previously determined in the members of the lower part and a Williot diagram was drawn, assuming Q and S (Fig. 2) as fixed in position. Transverse moments of inertia were computed and the moments at the ends of each member expressed in the form of Equations (1) to (3), except that ψ for each member was given the numerical value determined from the Williot diagram. The wind forces on the structure are shown acting from right to left. The moments of these forces above and about any section are counter-clockwise and in this paper are called positive, hence the counter-clockwise couples, M, and the counter clockwise rotations, ϕ and ψ, are positive.

The equations, $\Sigma M = 0$, were written for Joints K, W, O, X, and Q. Since ϕ has the same value for corresponding joints; that is, $\phi_l = \phi_k$, etc., only the two diagonal members on the left of the center line of the tower need be considered in the equations, $\Sigma M = 0$, for Joints W and X. In the equation for Joint K, M_{ki} was not expressed in terms of ϕ_i and ϕ_k, but was equated to the known quantity found at the beginning of the second approximation, for by so doing ϕ_i was eliminated from the equation. The five equations were solved and the angles, ϕ, determined.

In the treatment of the upper portion, M_{ik} and M_{ki} were expressed in terms of ϕ_i, ϕ_k, and ψ_{ik}, and each was equated to its approximate value as determined in the second approximation. The known quantity obtained in the analysis of the lower portion was substituted for ϕ_k and the two equations were solved for ϕ_i and ψ_{ik}. The remaining members of the post were similarly treated, and the quantities, ϕ, for each joint and, ψ, for each member determined. From the known values of ψ the transverse moments of the eccentric dead and live loads were computed, and the sum, M_{ak} of these moments from A to K was determined. Since the wind stresses were tensile in one post and compressive in the other, their eccentric moments balanced each other and, therefore, were not included.

Heretofore all computations had been based on the outline sketch of Fig. 2. A drawing to scale was now made of the transverse view. The horizontal struts above the floor were assigned vertical dimensions, giving a proper architectural balance to the structure, as outlined in Fig. 3. When the bracing below the floor was added, it was evident that, for good detailing, the diagonals should intersect the horizontals at the inside faces of the posts rather than on their center lines. These eccentric connections had also a theoretical advantage, since the resulting eccentric moments reduced the lateral deflection of the tower, The member, QS, was raised to a proper distance from the base and a new post member, QT, entered the problem.

The computations for this second tentative design had been made on the assumption that the shear in each panel below the floor was resisted entirely by the web system. The process by which this assumption was eliminated, namely, by analyzing the wind shear distribution between the posts and bracing, will now be explained. Throughout the analysis the following symbols with identification subscripts will be used. All dimensions are expressed in inches; all loads and stresses, in pounds; and all moments, in inch-pounds:

b = distance center to center of posts.
W = dead and live load stress in any post member.
H = horizontal shear in any post member, or the horizontal component of stress in any diagonal below the floor.
V = vertical component of stress in any diagonal below the floor, or the vertical shear in any strut above the floor.
P = axial wind stress in any member.
l = length of any member.
$\Delta = \frac{Pl}{AE}$ = the wind strain in any member.
S = one-half the total wind shear in any panel.
M_{ak} = sum of the transverse eccentric moments in one post from A to K.
M_w = moments of wind forces on one post above and about W.
M_x = moments of wind forces on one post above and about X.
M_T = moments of wind forces on one post about the center line of the tower at the base.

From Fig. 5,

$$H_{ko} + H_{kw} = S_{ko} \quad \ldots\ldots(4)$$

also,

$$M_{ko} + M_{ok} + l_{ko}\,\psi_{ko}\,W_{ko} + l_{ko}\,H_{ko} = 0 \quad \ldots\ldots(5)$$

or,

$$M_{ko} + M_{ok} + l_{ko}\,\psi_{ko}\,W_{ko} = l_{ko}\,(H_{kw} - S_{ko}) \quad \ldots\ldots(6)$$

The quantities, M_{ko} and M_{ok}, may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}, while l_{ko}, W_{ko}, and S_{ko} are known; hence, H_{kw} may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. As H_{kw} is the horizontal component of the stress in KW and KWR, the strains in these members, Δ_{kw} and Δ_{kwn}, may be

expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. Likewise, Δ_{ox} and Δ_{oxs} may be expressed in terms of ϕ_o, ϕ_q, and ψ_{oq}.

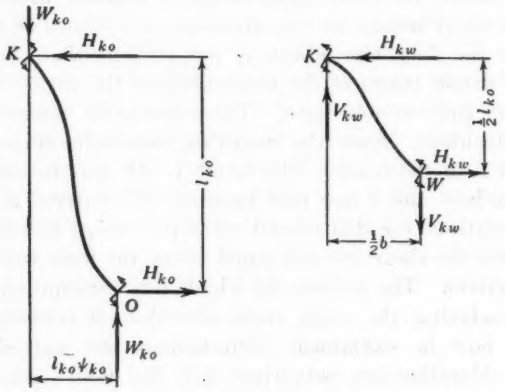

FIG. 5.

The stress in KO may be found by balancing the moments about W and observing that the moment at the middle of the member, KO, is $\frac{M_{ok} - M_{ko}}{2}$, whence,

$$\frac{b}{2} P_{ko} = M_W + M_{ak} + \frac{l_{ko}}{2} \psi_{ko} W_{ok} + \frac{M_{ok} - M_{ko}}{2} \quad \ldots\ldots\ldots (7)$$

from which, Δ_{ko} may be expressed in terms of ϕ_k, ϕ_o, and ψ_{ko}. Also,

$$\frac{b}{2} P_{oq} = M_X + M_{ak} + l_{ko} \psi_{ko} W_{ko} + \frac{l_{oq}}{2} \psi_{oq} W_{oq} + \frac{M_{qo} - M_{oq}}{2} \quad \ldots. (8)$$

from which, Δ_{oq} may be expressed in terms of ϕ_o, ϕ_q, ψ_{ko}, and ψ_{oq}. Likewise,

$$\frac{b}{2} P_{qt} = M_T + M_{ak} + l_{ko} \psi_{ko} W_{ko} + l_{oq} \psi_{oq} W_{oq} + l_{qt} \psi_{qt} W_{qt} + M_{tq} \quad \ldots. (9)$$

from which, Δ_{qt} may be expressed in terms of ϕ_q, ψ_{ko}, ψ_{oq}, and ψ_{qt}.

The equations, $\Sigma M = 0$, were written for Joints K, W, O, X, and Q; also, the equation,

$$M_{tq} + M_{qt} + l_{qt} \psi_{qt} W_{qt} + l_{qt} S_{qt} = 0 \quad \ldots\ldots\ldots\ldots\ldots (10)$$

a total of six equations.

A moment term, M_k, to account for the eccentric diagonal connection was included in the equation, $\Sigma M = 0$, for Joint K, as follows: Let a equal one-half the width of the post at K, then $M_k = aV_{kw}$. From Fig. 5:

$$M_{kw} + M_{wk} + \frac{1}{2} l_{ko} H_{kw} = \frac{1}{2} b V_{kw} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

whence M_k may be expressed in terms of ϕ_k, ϕ_w, and ψ_{kw}. The eccentric moments at O and Q are treated in a similar manner.

In the equation for Joint K a correction is made in the value of M_{ki}, which was previously determined as indicated in the first paragraph of the second approximation by assuming a point of contraflexure at the mid-height of the panel, or, in other words, by making $M_{ik} = M_{ki}$ in the equation, $M_{ik} + M_{ki} + l_{ik} S_{ik} = 0$.

After the value of M_{ki} had thus been obtained, the panel height, l_{ik}, was changed by assigning a depth to Strut 4; and also an approximate value for ψ_{ik} was determined, as described in the third paragraph of this third approximation. Therefore, the term, $l_{ik}\ \psi_{ik}\ W_{ik}$, representing the moment of the eccentric dead and live loads may now enter the computations. Hence, from Fig. 1,

$$M_{ik} + M_{ki} = - l_{ik} S_{ik} - l_{ik} \psi_{ik} W_{ik} \ldots\ldots\ldots\ldots (12)$$

From Equation (12) it is obvious that the sum, $M_{ik} + M_{ki}$, is statically determinate whatever the design, except for the quantity, ψ_{ik}, which is so comparatively small that any variation in its magnitude will have slight effect on the sum. For example, the magnitude of ψ_{ik} was found to be approximately 0.0017; hence,

$$\begin{aligned} - l_{ik}\ S_{ik} &= - 1\,476 \times 1\,715\,000 = - 2\,531\,000\,000 \text{ in-lb} \\ - l_{ik} \psi_{ik} W_{ik} &= - 1\,476 \times 0.0017 \times 74\,000\,000 = - 186\,000\,000 \text{ in-lb} \\ \hline M_{ik} + M_{ki} &= - 2\,717\,000\,000 \text{ in-lb} \end{aligned}$$

Instead of dividing this sum equally it was decided to divide it proportionately to the section moduli of the post at I and K, so that the resulting unit stresses would be equal, with the following result;

$$\begin{aligned} M_{ik} &= - 1\,291\,000\,000 \text{ in-lb} \\ M_{ki} &= - 1\,426\,000\,000 \text{ in-lb} \\ \hline M_{ik} + M_{ki} &= - 2\,717\,000\,000 \text{ in-lb} \end{aligned}$$

The magnitude of M_{ki} was thus arbitrarily fixed at 1 426 000 000 in-lb, the design was made to conform thereto, and any variation in the sum of the two moments on account of later changes in ψ_{ik} was absorbed, therefore, in M_{ik}. The term, M_{ki}, is expressed numerically in the equation, $\Sigma M = 0$, for Joint K; therefore, the quantities, ϕ_i and ψ_{ik}, do not appear in the equation.

The post is assumed to be completely restrained at T; that is, $\phi_t = 0$. Hence, the unknowns appearing in the six equations are a ϕ for each of five joints and a ψ for each of ten members, making fifteen unknowns. Nine additional equations are necessary therefore, for a solution. These nine equations may be obtained from the geometrical properties of Fig. 6. This is a Williot diagram as ordinarily understood with certain basic variations. Usually such a diagram can be made only when the strains corresponding to Δ_{qx} and the other heavy lines on Fig. 6 are known. Actually, the lengths of these lines are unknown; but their directions are known. Hence, the Williot diagram can be sketched and from its geometrical properties relations can be set up between the various strains in the form of equations.

Letting θ (Fig. 6) indicate the angle that the diagonal makes with the vertical, the nine required equations will then be as follows:

$$l_{qx}\,\psi_{qx} = \Delta_{qt}\,\text{cosec}\,\theta + \Delta_{qx}\cot\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$l_{ox}\,\psi_{ox} = (\Delta_{qt} + \Delta_{oq})\,\text{cosec}\,\theta + \Delta_{ox}\cot\theta \quad \ldots\ldots\ldots\ldots (14)$$

$$l_{ow}\,\psi_{ow} = (\Delta_{qt} + \Delta_{oq})\,\text{cosec}\,\theta + \Delta_{ow}\cot\theta \quad \ldots\ldots\ldots\ldots (15)$$

$$l_{kw}\,\psi_{kw} = (\Delta_{qt} + \Delta_{oq} + \Delta_{ko})\,\text{cosec}\,\theta + \Delta_{kw}\cot\theta \quad \ldots\ldots\ldots (16)$$

$$l_{oq}\,\psi_{oq} = (2\,\Delta_{qt} + \Delta_{oq})\cot\theta + \Delta_{oxs}\,\text{cosec}\,\theta \quad \ldots\ldots\ldots\ldots (17)$$

$$l_{ko}\,\psi_{ko} = (2\,\Delta_{qt} + 2\,\Delta_{oq} + \Delta_{ko})\cot\theta + \Delta_{kwr}\,\text{cosec}\,\theta \quad \ldots\ldots\ldots (18)$$

$$b\,\psi_{qs} = 2\,\Delta_{qt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$$b\,\psi_{or} = 2\,(\Delta_{qt} + \Delta_{oq}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

$$b\,\psi_{kl} = 2\,(\Delta_{qt} + \Delta_{oq} + \Delta_{ko}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

The expression previously derived for each Δ, in terms of ϕ and ψ, may now be substituted in these nine equations, and the fifteen equations solved for the fifteen unknown quantities, ϕ and ψ.

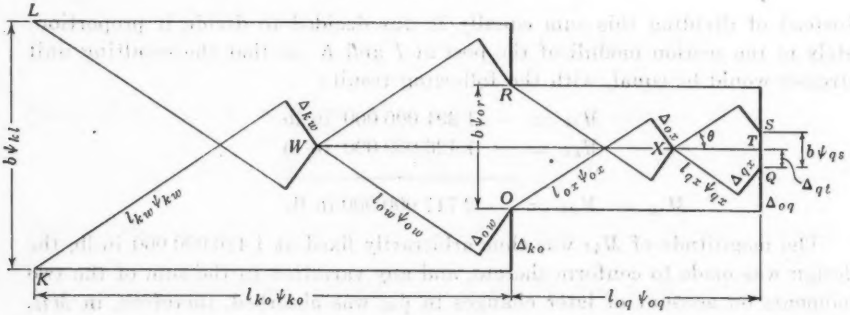

FIG. 6.

In this particular design, the diagonals in the two panels had equal areas since the wind shear differed but slightly in the two panels, one-half the shear being 2 767 000 lb in the upper panel and 2 979 000 lb in the lower panel. When the equations had been solved and the stresses, moments, and shears computed, it was found that the diagonal, *KWR*, was carrying a shear of 3 162 000 lb and resisting not only the wind shear but also the 395 000-lb shear in the post, *KO*, as well; while the diagonal, *OXS*, was carrying a shear of only 1 363 000 lb, the remaining 1 616 000 lb being resisted by the post, *OQ*. The moment, M_{ko}, was nearly double the moment, M_{ok}, while the moment, M_{oq}, was only about one-tenth the moment, M_{qo}, and a point of contraflexure still remained in this member. The stress in *KWR* was considerably more than double the stress in *OXS*.

These results were rather astonishing at first sight, but a numerical check showed that the moments, shears, and stresses balanced for each joint and section. The computations clearly indicated that a better balance in shear distribution should be sought and that strain rather than strength should be the governing factor in the design of the web system. Shifting from one system of bracing to another in the middle of a structure is not unlike changing commanders in the midst of a battle or swapping horses in the middle of a stream. Radical changes in the diagonals and minor changes in the posts were necessary before a satisfactory balance in shear distribution was secured, each set of changes being followed by corresponding revisions in the fifteen equations which were then solved for the fifteen quantities, ϕ and ψ.

In the treatment of the upper portion, M_{ki} was expressed in terms of ϕ_k, ϕ_i, and ψ_{ik}, and equated to its fixed magnitude. In Equation (12) the known magnitude of M_{ki} was substituted and M_{ik} expressed in terms of ϕ_k, ϕ_i, and ψ_{ik}. The quantity, ϕ_k, was known, having been determined from the computations made for the lower portion after the last revision mentioned in the preceding paragraph. The two equations were solved for ϕ_i and ψ_{ik} and the moment, M_{ik}, was computed. The sum, $M_{jg} + M_{gj}$, was then deter-

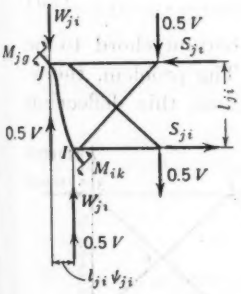

FIG. 7.

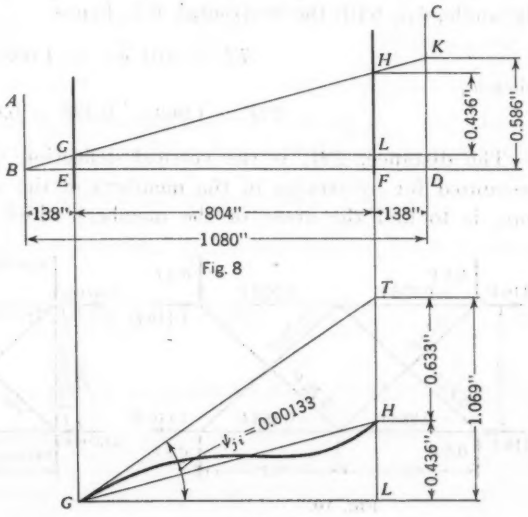

Fig. 8

FIG. 9.

mined from an equation similar to Equation (12), the sum divided proportionately to the transverse section moduli of the post at J and G, and a definite magnitude fixed for M_{jg}.

If a point of contraflexure is assumed at the middle of Strut 4 then, from Fig. 7, if V is the vertical shear in the strut,

$$M_{ik} + M_{jg} + l_{ji}\,\psi_{ji}\,W_{ji} + l_{ji}\,S_{ji} = \frac{1}{2}\,bV \ldots\ldots\ldots\ldots (22)$$

All quantities in the first member of Equation (22) are known except ψ_{ji}, which in the first approximation of the strut may be assumed equal to ϕ_i and the shear, V, determined.

An interesting problem arises at this point. The following quantities are known: $V = 5\,138\,000$ lb; $\psi_{ji} = \phi_i = 0.00133$; $\Delta_{it} = \Delta_{ik} + \Delta_{ko} + \Delta_{oq} + \Delta_{qt} = 0.293$ in.; and $P_{ik} = 13\,269\,000$ lb. The strut must be designed so that, when rotated through the angle, ψ_{ji}, and then distorted by the shear, V, the elevations of the ends of the bottom chord will be consistent with the strains, Δ_{it}, in the posts below. In Fig. 8, let AB and CD represent the center lines of the posts; GE and HF, their inside faces; and EF the center line of the bottom chord of the strut—all in their unstrained positions. The strain in each post is $\Delta_{it} = 0.293$ in., shortening one post and lengthening the other so that the difference in elevations of B and D after distortion is $0.293 \times 2 = 0.586$ in. If this difference is represented by KD, then the difference in elevations of the two ends of the bottom chord is,

$$HL = \frac{804}{1080} \times 0.586 = 0.436 \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

In Fig. 9, GT is the tangent to the elastic curve of the bottom chord making the angle, ψ_{ji}, with the horizontal, GL, hence,

$$TL = 804\ \psi_{ji} = 1.069 \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

whence,

$$TH = 1.069 - 0.436 = 0.633 \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The distance, TH, is the vertical deflection of the bottom chord to be accounted for by strains in the members of the strut. The problem, therefore, is to find the areas of the members which will cause this deflection

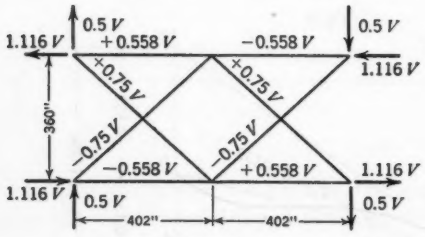

FIG. 10.

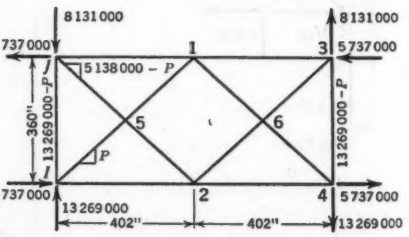

FIG. 11.

when the members are stressed by a shear, $V = 5\,138\,000$ lb. The stresses, S, in terms of V, divided equally between the two web systems are given in Fig. 10.

Let A = the area of all the members, then,

$$TH = \sum \frac{S\,u\,l}{A\,E} = \frac{0.000\,059\,V}{A} = \frac{303}{A} = 0.633 \ldots\ldots\ldots\ldots (26)$$

whence, $A = 478$ sq in.

This area gives low unit stresses—about 6 000 lb for the chords and 8 000 lb for the webs—which indicates that small strains or stiffness rather than strength governs the design. Since stiffness is required, the members themselves as well as the strut as a whole should be designed with this end in view, by concentrating as much metal in the extreme fibers or flanges as is consistent with good design.

The distribution of the shear between the two web systems, *I*-1-4 and *J*-2-3 (Fig. 11) will now be investigated. Let P represent the vertical component of stress in *I*-1, and $5\,138\,000 - P$, the vertical component in *J*-2. The stresses, S, and the strains, Δ, multiplied by E, are given in Table 1.

TABLE 1.—WIND STRESSES AND STRAINS MULTIPLIED BY E IN STRUT 4, THIRD APPROXIMATION

Member	l	A	$\frac{l}{A}$	S	$E\,\Delta = \frac{Sl}{A}$
I-2.....	402	478	0.841	$5\,737\,000 - 1.116\,P$	$4\,825\,000 - 0.939\,P$
2-4.....	402	478	0.841	$5\,737\,000 - 1.116\,P$	$4\,825\,000 - 0.939\,P$
J-1.....	402	478	0.841	$1.116\,P$	$0.939\,P$
1-3.....	402	478	0.841	$1.116\,P$	$0.939\,P$
I-1.....	540	A	a	$1.5\,P$	$1.5\,a\,P$
1-4.....	540	A	a	$1.5\,P$	$1.5\,a\,P$
J-2.....	540	478	1.13	$7\,702\,000 - 1.1499\,P$	$8\,701\,000 - 1.693\,P$
2-3.....	540	478	1.13	$7\,702\,000 - 1.1499\,P$	$8\,701\,000 - 1.693\,P$
J-*I*.....	360	7 220	0.0498	$13\,269\,000 - P$	$662\,000 - 0.0498\,P$
3-4.....	360	7 220	0.0498	$13\,269\,000 - P$	$662\,000 - 0.0498\,P$

The post members, *J*-*I* and 3-4, have an area of 7 220 sq in. The areas of all strut members are 478 sq in., except the diagonals, *I*-1 and 1-4, which are assigned an unknown area, A, and an unknown value of $\frac{l}{A} = a$.

In Fig. 9, let $T\,H = d =$ the vertical deflection of Joint 4, due to strains in the members, then,

$$d = 804\,\psi_{ji} - 0.436 \qquad (27)$$

or,

$$E\,d = 29\,000\,000\,(804\,\psi_{ji} - 0.436) = 23\,316\,000\,000\,\psi_{ji} - 12\,655\,000 \qquad (28)$$

Two general expressions for d in terms of the strains may be obtained from the Williot diagram, Fig. 12. Joint *I* is fixed in position; Member *J*-*I*, fixed in a vertical direction; and the vertical deflection of Joint 4 is *I*-4. If $\theta =$ the angle, 1-*I*-2, then,

$$E\,d = E\,(\Delta_{j2} + \Delta_{24})\,\operatorname{cosec}\theta + E\,(\Delta_{i2} + \Delta_{24})\,\cot\theta + E\,(\Delta_{ji} + \Delta_{34}) \qquad (29)$$

also,

$$E\,d = E\,(\Delta_{i1} + \Delta_{14})\,\operatorname{cosec}\theta + E\,(\Delta_{j1} + \Delta_{13})\,\cot\theta \qquad (30)$$

Equating the two expressions for $E\,d$ in Equations (28) and (29), substituting the expressions for the strains, Δ, from Table 1, and solving,

$$P = -\,3\,205\,000\,000\,\psi_{ji} + 6\,989\,000 \qquad (31)$$

A Williot diagram, Fig. 13, may be drawn to determine an approximate value of ψ for each strut member. The stresses are computed on the assumption that the shear of 5 138 000 lb is resisted equally by the two web systems. The strains are determined therefrom by assuming 478 sq in. as the area of all members. Joint I is fixed in position and Member J-I is rotated through the angle, $\psi_{ji} = 0.00133$; whence the horizontal distance between I and J is $360\ \psi_{ji} = 0.479$ in. Then the vertical distance, 4-I, which corresponds to

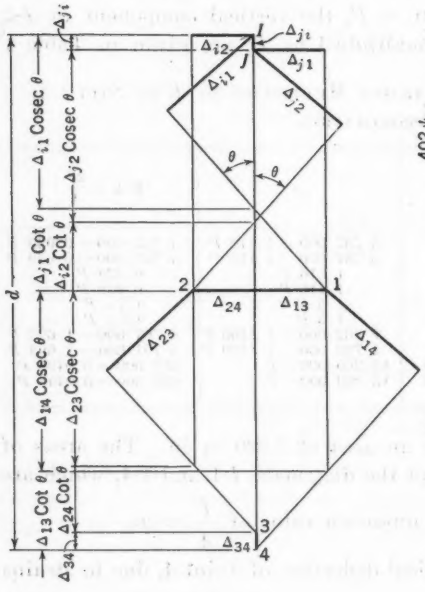

FIG. 12.

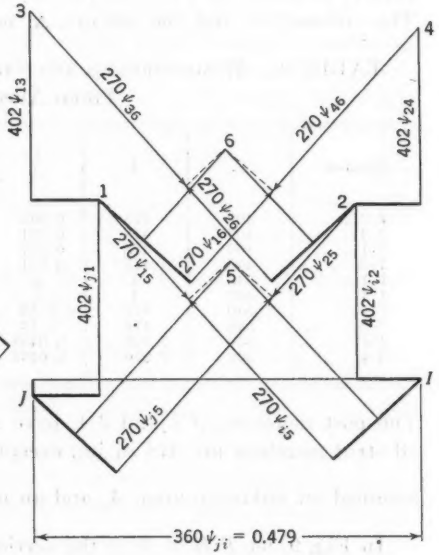

FIG. 13.

$H\ L$ in Fig. 9, scales approximately 0.436 in. In scaling the quantities, $l\psi$, they are found to be approximately symmetrical about the center line of the strut and the values of ψ are thus assumed. Exact instead of approximate values of ψ for the strut members could have been included in the computations by simply sketching Fig. 13 and developing therefrom the expression for each ψ in terms of ψ_{ji} and the strains, Δ, as given in Table 1. The expedient used, however, saved considerable labor at a small sacrifice in accuracy.

The moment at the end of each member in the left half of the strut is expressed in terms of ϕ and ψ and the known value of ψ is substituted. The equations, $\Sigma M = 0$, are written for Joints I, 1, 2, and 5. At Joints 1 and 2, ΣM includes only the two members on the left of the center line. In the equation for Joint I there will be a term, M_i, to account for the eccentric connection of the diagonal, thus,

$$M_i = 138\ P = 138\ (-\ 3\ 205\ 000\ 000\ \psi_{ji} + 6\ 989\ 000) \ldots\ldots (32)$$

A fifth equation is written,

$$M_{ji} + M_{ij} + l_{ji} S_{ji} + l_{ji} \psi_{ji} W_{ji} - 402V = 0 \ldots\ldots (33)$$

Since ϕ_i is known, the five equations may be solved for ϕ_j, ϕ_1, ϕ_2, ϕ_5, and ψ_{ji}. Then the first four terms in the equation,

$$M_{ji} + M_{j5} + M_j + M_{j1} + M_{jg} = 0 \ldots\ldots\ldots\ldots (34)$$

may be computed and M_{jg} determined.

The magnitude of M_{jg} thus found differed considerably with the desired value previously assigned, since in approximating the shear, V, from Equation (22), ψ_{ji} was assumed equal to ϕ_i and no account was taken of the moments at the ends of the strut members at J and I. The value of ψ_{ji} was corrected in Equations (22), (24), and (25); the end moments of the four strut members were included in Equation (22); and a revised value of the shear, V, was obtained. When the computations had been revised a satisfactory agreement existed between the derived and assigned magnitudes of M_{jg} and no further revision was necessary.

The value of P, determined from Equation (31), was substituted in the expressions for $E\Delta$ in Table 1 and $a = \dfrac{l}{A}$ was determined from Equation (30), from which the required area of the diagonals, I-1 and 1-4, was found. The remaining struts were treated in a similar manner.

Fourth Approximation

In the analysis of Strut 1 no variation in the shear, V, could be made, since $V = P_{cd}$. When the analysis had been completed the moment, M_{ba}, did not check with the moment of the known forces above B. This was accounted for by the fact that in Equations (7), (8), and (9) the eccentric moments, M_{ak}, were taken from a rough approximation to the elastic curve and no account was made of the moments at the ends of the sixteen strut members. The equations were corrected accordingly and the computations revised throughout.

The corrections for the lower portion were quite simple and, because no changes in design were necessary, were confined to the numerical terms in the equations affected. Revisions in design were necessary only in Struts 1 and 2. The value of M_{ba} was then in close agreement with the known value.

The expedient of assigning definite magnitudes to the moments M_{ki}, M_{jg}, M_{fe}, and M_{dc}, and making the design conform thereto greatly reduced the amount of work. Any set of computations and revisions was then confined to a relatively small part of the structure.

Further attention was now given to the longitudinal moments. In the second approximation a parabola was assumed for the elastic curve, tangent at the base with a deflection of 17 in. at the top. In the fourth approximation, x_1 and x_2 were taken as the deflections at K and F, and the deflections at all other points where there was a change in section were expressed as func-

tions of the unknowns, x_1 and x_2. Expressions in terms of x_1, x_2, and H were developed by the area-moment method for the deflections at K, F, and the top, and were equated to x_1, x_2 and 17 in., respectively. The equations were solved, the remaining deflections determined, and the longitudinal moments computed. In the second approximation the deflections at F and K were 7.922 and 1.313 in., respectively, as computed from the parabola. In the analysis just described the deflections were found to be 7.631 and 1.175 in., respectively.

Review of the Design

Since the design had been made by fixing arbitrarily the magnitude of the moments, M_{ki}, M_{jg}, M_{fe}, and M_{dc}, and designing the structure accordingly, it was decided to make an independent set of computations, beginning

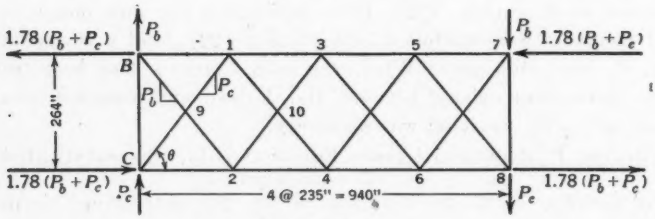

FIG. 14.

in this case at the top. The following outline gives the gist of the method in greatly condensed form. The brief details should help other designers to a direct, time-saving solution.

TABLE 2.—WIND STRESSES AND STRAINS MULTIPLIED BY E IN STRUT 1, REVIEW COMPUTATIONS

Member	l	A	$\frac{l}{A}$	S	$E\Delta = \frac{Sl}{A}$
C-2	235	452.5	0.519	1.78 P_b + 0.89 P_c	0.925 P_b + 0.462 P_c
2-4	235	452.5	0.519	0.89 P_c	0.462 P_c
4-6	235	452.5	0.519	0.89 P_c	0.462 P_c
6-8	235	452.5	0.519	1.78 P_b + 0.89 P_c	0.925 P_b + 0.462 P_c
B-1	235	452.5	0.519	0.89 P_b + 1.78 P_c	0.462 P_b + 0.925 P_c
1-3	235	452.5	0.519	0.89 P_b	0.462 P_b
3-5	235	452.5	0.519	0.89 P_b	0.462 P_b
5-7	235	452.5	0.519	0.89 P_b + 1.78 P_c	0.462 P_b + 0.925 P_c
C-1	353	463	0.763	1.339 P_c	1.022 P_c
1-4	353	463	0.763	1.339 P_c	1.022 P_c
4-5	353	463	0.763	1.339 P_c	1.022 P_c
5-8	353	463	0.763	1.339 P_c	1.022 P_c
B-2	353	452.5	0.781	1.339 P_b	1.046 P_b
2-3	353	452.5	0.781	1.339 P_b	1.046 P_b
3-6	353	452.5	0.781	1.339 P_b	1.046 P_b
6-7	353	452.5	0.781	1.339 P_b	1.046 P_b
B-C	264	3 895	0.068	P_b	0.068 P_b
7-8	264	3 895	0.068	P_b	0.068 P_b

Let P_b and P_c represent the vertical components of the stress in the diagonals of Strut 1, as shown in Fig. 14. The computations of stresses and strains were made as in Table 2. Let d = the vertical deflection of Joint 8

due to strains, and Δ_{ct} = strain in the post from C to T. Then,

$$d = 940\,\psi_{bc} - \left(2\,\Delta_{ct} \times \frac{940}{1080}\right)$$

or,

$$E\,d = 27\,260\,000\,000\,\psi_{bc} - 50\,481\,000\,\Delta_{ct} \ldots\ldots\ldots\ldots (35)$$

The following equations were derived from the geometry of a Williot diagram drawn similar to Fig. 12:

$$E\,d = 4\,E\,\Delta_{b2}\,\text{cosec}\,\theta + E\,(4\Delta_{c2} + 2\,\Delta_{b1} + 2\,\Delta_{13})\,\cot\theta + E\,(\Delta_{bc} + \Delta_{73}) \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$E\,d = 4\,E\,\Delta_{c1}\,\text{cosec}\,\theta + E\,(4\,\Delta_{b1} + 2\,\Delta_{c2} + 2\,\Delta_{24})\,\cot\theta \ldots . (37)$$

From Equations (36) and (37) P_b and P_c were expressed in terms of ψ_{bc} and Δ_{ct}. Eleven equations were written—one each for M_{ab} and M_{ba}; one each for Joints B, C, 1, 2, 3, 4, 9, and 10; and one for $M_{bc} + M_{cb}$. These eleven equations contained fourteen unknown quantities, ϕ_a, ϕ_b, ϕ_c, ϕ_d, ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_9, ϕ_{10}, ψ_{ab}, ψ_{bc}, ψ_{cd}, and Δ_{ct}. The quantities, ψ, for the strut members were not treated as unknowns; their magnitudes were obtained from the Williot diagram drawn similar to Fig. 13 after the final revision in the design had been made. Five of these fourteen unknowns, ϕ_c, ϕ_d, ψ_{bc}, ψ_{de}, and Δ_{ct}, appear in later equations. The remaining nine were eliminated from the eleven equations, leaving two equations containing the five unknowns, ϕ_c, ϕ_d, ψ_{bc}, ψ_{de}, and Δ_{ct}.

A similar treatment was given to Strut 2 which is 888 in. long. Then, as before,

$$d = 888\,\psi_{de} - \left(2\,\Delta_{ct} \times \frac{888}{1080}\right)$$

Two equations were developed from the geometry of a Williot diagram and a third equation was written as follows: $\Delta_{et} = \Delta_{ct} - \Delta_{cd} - \Delta_{de}$. From these three equations the vertical components in the diagonals, P_d, and P_e, were expressed in terms of ψ_{bc}, ψ_{de}, and Δ_{ct}. Eight equations, $\Sigma M = 0$, were written for the eight joints, one for $M_{cd} + M_{dc}$ and one for $M_{de} + M_{ed}$. These equations, with the two brought down from Strut 1, gave twelve equations from which nine unknowns were eliminated, leaving three equations having the six unknowns, ϕ_e, ϕ_f, ψ_{bc}, ψ_{de}, ψ_{ef}, and Δ_{ct}.

Strut 3 was similarly treated, resulting in four equations having seven unknowns, ϕ_g, ϕ_j, ψ_{bc}, ψ_{de}, ψ_{fg}, and Δ_{ct}. The analysis of Strut 4 gave five equations with eight unknowns, ϕ_i, ϕ_k, ψ_{bc}, ψ_{de}, ψ_{fg}, ψ_{ji}, ψ_{ik}, and Δ_{ct}.

An equation was now developed by equating two independent expressions for Δ_{kt} as follows:

$$\Delta_{kt} = \Delta_{ct} - (\Delta_{cd} + \Delta_{de} + \Delta_{ef} + \Delta_{fg} + \Delta_{gj} + \Delta_{ji} + \Delta_{ik}) = \Delta_{ko} + \Delta_{oq} + \Delta_{qt}$$

From these six equations, ψ_{bc}, ψ_{de}, ψ_{fg}, ψ_{ji}, and Δ_{ct}, were eliminated, leaving

one equation having the following unknowns: ϕ_i, ϕ_k, ϕ_o, ϕ_q, ψ_{ik}, ψ_{ko}, ψ_{oq}, and ψ_{qt}. An equation in terms of ϕ_i, ϕ_k, and ψ_{ik}, was written from $M_{ik} + M_{ki}$. The lower portion was treated in the same way as in the fourth approximation, except that M_{ki} was expressed in terms of ϕ_i, ϕ_k, and ψ_{ik} instead of as a known quantity. This introduced two additional unknowns, ϕ_i and ψ_{ik}, into the equations for the lower portion, but the upper portion, in turn, supplied the two additional equations necessary for the solution.

All assumptions made in the approximate solutions have been eliminated except that of identical elastic curves for the two posts. Two possible exceptions to this statement may be claimed. One exception is the assumption of numerical terms in Equations (7), (8), and (9) for the sum of the moments, M_s, in the sixteen strut members and for the sum of the eccentric moments, M_{ak}. The error involved is the difference between the magnitudes appearing in the equations and the magnitudes derived therefrom.

For example, in the final review of the problem, the magnitudes used in the equations were $M_{ak} = +\ 665\,000\,000$ in-lb and $M_s = -\ 239\,000\,000$ in-lb, as determined from the fourth approximation. When the review was completed the magnitudes were found to be $M_{ak} = +\ 663\,000\,000$ in-lb and $M_s = -\ 235\,000\,000$ in-lb. In any given set of computations, M_{ak} and M_s, are, respectively, the same in all three Equations (7), (8), and (9); and when these are combined with either $M_w = +\ 9\,707\,000\,000$ in-lb in Equation (7), or with $M_x = +\ 12\,930\,000\,000$ in-lb in Equation (8), or with $M_r = +\ 14\,874\,000\,000$ in-lb in Equation (9), the errors are too small for consideration.

The other exception is the assumption that the post bases are in complete restraint against rotation, or $\phi_t = 0$. In order to ascertain the total possible range of moments, shears, and stresses, a review was also made on the assumption that the base was free to rotate by making $M_{tq} = 0$ and treating ϕ_t as an unknown quantity. The chief differences in the results of these two assumptions were confined to the part below the floor. For the fixed base, the moments in the posts and the stress in the upper diagonals were larger, and the stress in the lower diagonals smaller, than for the free base. The transverse deflection at the top was 11.14 in. for the fixed assumption and 11.31 in. for the free assumption.

Conclusion

It seems unnecessary to make extended comments or to draw specific conclusions from this expository rather than argumentative monograph, further than to emphasize the simplicity of the process by which the Williot equations are developed. The facility of their application, however, will vary according to the nature and degree of complexity of the problem in hand. There is, of course, an analogy between the Williot deflection equations and the energy or elastic equations derived from equating external to internal work. When confronted with the analysis of any statistically indeterminate structure, the analyst will solve the problem by the method with which he is the most familiar, whether it is the elastic, the deflection, or any other method.

provided the particular method adopted ends in fruition. All methods, however, are not equally efficient. In the present case the completed analysis of the towers may be checked by equating internal and external work after the problem has been solved and the deflections are known; but the problem cannot be solved by this method except possibly by an almost endless procession of "cut and try" guesses as to the displacements of all joints in the structure; whereas by the method of expressing moments in terms of angular displacements, the displacements may be treated as unknowns and a solution obtained with the help of the Williot strain equations.

It may be that this method is known to others, but during his years of academic and professional experience the writer has never encountered a presentation of it, and probably never would have discovered the method had not necessity given it birth.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESS ANALYSIS SIMPLIFIED

Discussion

BY L. E. GRINTER, ASSOC. M. AM. SOC. C. E.

L. E. GRINTER,[34] ASSOC. M. AM. SOC. C. E. (by letter)[34a].—The constructive nature of the criticism apparent in nearly all the discussions of this paper is appreciated. The writer wishes particularly to thank Messrs. Letherbury and Reese for their carefully prepared discussions showing the application of the simplified method to the analysis of practical building frames. He is highly mindful of the labor involved in the preparation of these valuable discussions.

Professor Witmer's discussion refers primarily to the determination of the moments caused by column deformation. He states correctly that this effect can be evaluated with the effect of horizontal translation, but it should be noted that this procedure is possible only when the fixed-end column moments are obtained from a preliminary estimate of deflections and approximate column stresses are known.

Mr. Timby requests the answers to several questions which the writer attempted to answer in the original paper. Of course, it is not true that the preliminary location of points of contraflexure at the mid-heights of the columns, or the temporary assumption that the story deflection is proportional to the story height, carries through the entire analysis and vitiates the final results. These assumptions merely form a starting point in the analysis. The simplified method is based on successive corrections, and the final moments can be calculated to agree with any predetermined standard of accuracy.

Perhaps it is unfortunate that the word, "ideal," was used to describe a bent theoretically proportioned so as to deflect linearly. However, no suggestion was made that such a frame was desirable. In fact, its only purpose was to help the reader bridge the gap from the single-story bent to the tall building

NOTE.—The paper by L. E. Grinter, Assoc. M. Am. Soc. C. E., was presented at the meeting of the Structural Division, New York, N. Y., January 19, 1933, and published in January, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1933, by Messrs. Francis P. Witmer, Elmer K. Timby, N. A. Richards, John B. Letherbury, Frederick Martin Weiss, and Raymond C. Reese; May, 1933, by Messrs. A. Floris, Robins Fleming, J. D. Gedo, H. V. Spurr, and Johannes Skytte; and September, 1933, by John B. Wilbur, Assoc. M. Am. C. E.

[34] Prof., Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[34a] Received by the Secretary November 13, 1933.

frame. Mr. Timby seems to have misunderstood the purpose of this part of the paper and digresses into a comparison of the ideal frame with a mechanical model in which he was particularly interested.

The results of the analysis of this model by means of the Beggs deformeter by Messrs. Coultas and Lawton were used by both Mr. Timby and Mr. Wilbur in drawing certain rather important conclusions. This was particularly unfortunate since it can be shown that the Coultas and Lawton analysis of this model bent is radically in error.[34b]

TABLE 11.—Comparison of Column Moments for Coultas and Lawton Bent as Obtained by the Simplified Method, the Slope Deflection Method, and by Mechanical Analysis

Story	Coultas and Lawton solution (use of Beggs deformeter) (1)	Slope deflection analysis (story loads and average *K*-values) (2)	Simplified method (story loads and average *K*-values) (3)	Simplified method (distributed load and average *K*-values) (4)	Simplified method (story loads and varying cross-sections) (5)	Simplified method (distributed load and varying cross-sections) (6)
			Column *A* (Windward Column)			
6	+ 4 140	+ 3 600	+ 3 500	+ 1 000	+ 3 700	+ 1 200
	+ 6 810	+ 2 800	+ 2 700	+ 7 400	+ 2 700	+ 7 300
5	+15 180	+11 900	+11 800	+ 7 500	+11 600	+ 7 300
	+ 8 210	+10 600	+10 600	+14 700	+10 000	+14 000
4	+25 382	+15 900	+16 100	+12 300	+16 000	+12 300
	+22 128	+14 800	+14 700	+18 400	+15 600	+19 400
3	+26 540	+22 200	+22 300	+19 000	+21 400	+18 200
	+24 260	+20 200	+20 200	+23 500	+20 000	+23 400
2	+14 550	+27 700	+27 700	+24 200	+27 600	+24 200
	+33 820	+27 300	+27 300	+30 900	+28 900	+32 700
1	+18 135	+22 600	+22 800	+19 500	+20 400	+17 100
	+45 450	+43 100	+43 200	+45 500	+44 500	+47 000
			Column *B* (Center Column)			
6	— 200	+ 6 000	+ 6 200	+ 5 300	+ 6 200	+ 5 400
	+ 9 500	+ 5 900	+ 6 200	+ 5 500	+ 5 600	+ 5 000
5	0	+15 600	+15 700	+15 900	+16 100	+16 200
	+31 450	+14 100	+13 900	+13 900	+15 400	+15 600
4	—17 160	+32 000	+32 200	+32 200	+30 000	+30 200
	+68 900	+29 600	+29 500	+29 600	+29 600	+29 800
3	+41 230	+40 000	+39 700	+39 800	+40 000	+39 900
	+35 270	+37 100	+37 000	+37 000	+39 200	+39 100
2	+59 350	+53 700	+53 800	+53 800	+50 800	+51 100
	+ 6 450	+53 000	+53 000	+53 000	+51 800	+52 200
1	+42 600	+41 600	+41 200	+41 200	+40 000	+40 200
	+67 900	+67 500	+67 200	+67 200	+70 200	+70 700
			Column *C* (Leeward Column)			
6	— 2 560	+ 3 600	+ 3 500	+ 3 300	+ 3 700	+ 3 400
	+ 2 185	+ 2 800	+ 2 700	+ 2 400	+ 2 700	+ 2 400
5	+10 900	+11 900	+11 800	+11 900	+11 600	+11 700
	+ 9 460	+10 600	+10 600	+10 600	+10 000	+10 000
4	+12 890	+15 900	+16 100	+16 100	+16 000	+16 100
	+11 678	+14 800	+14 700	+14 700	+15 600	+15 700
3	— 4 800	+22 200	+22 300	+22 100	+21 400	+21 400
	+40 800	+20 200	+20 200	+20 300	+20 000	+20 000
2	+33 700	+27 700	+27 700	+27 700	+27 600	+27 800
	+68 500	+27 300	+27 300	+27 300	+28 900	+29 000
1	+24 000	+22 600	+22 800	+22 800	+20 400	+20 500
	+39 010	+43 100	+43 200	+43 200	+44 500	+44 800

The Coultas and Lawton bent was intended to represent a practical six-story building frame. Hence, the columns change section above the girder line, and the girders are reduced in cross-section near the columns. Thus,

[34b] Special copies of the author's proof will be mailed to inquirers on request.

the analysis is complicated by intra-panel change in cross-section as well as by the fact that the wind force is taken as distributed over the windward column. An approximate analysis can readily be made by the simplified method by concentrating the loads at the floor levels and by using average K-values to represent the stiffness factors of the members.

The true carry-over factors and the distribution factors at the joints will be affected by intra-panel changes in cross-section. In order to obtain a more exact analysis than would result from average K-values, the actual factors should be used. Furthermore, the effect of the distributed loading in producing additional fixed-end moments in the windward column must be considered. Such analyses were performed, but are not included herein. It is sufficient to state that a comparison of column moments determined by four such analyses with those determined by Messrs. Coultas and Lawton, are given in Table 11 and show the latter to be so radically different from any of the others that one can only conclude that they are entirely valueless.

Since Mr. Timby's further observations and questions were based upon his study of this incorrect deformeter analysis, it seems unnecessary to take further space for their consideration. Mr. Wilbur observed that the deformeter analysis must take incorrectly into account the effect of direct stress and shearing deformation, but he was properly not convinced that this effect plus the effect of the distributed load could account for the unusual results shown in his Table 10. Table 11 shows the effect of load distribution and of intra-panel change in cross-section, and deflections measured on a model by Mr. M. P. Frank (formerly Instructor in the Civil Engineering Department of the Agricultural and Mechanical College of Texas, College Station, Tex.), prove that the effect of direct stress and shearing deformations are not serious for this particular bent. Lateral deflections computed by different methods were determined by Mr. Frank as shown in Fig. 18, in which Curve 5 represents the deflections computed from final moments obtained by the simplified method, using distributed loading and members with varying cross-sections. The deflection curve computed from the moments obtained by the slope-deflection method is identical with Curve 3, Fig. 18, of estimated deflections.

In regard to load distribution, the writer would like to emphasize the point that actual wind loading is not distributed entirely to the column. In fact, where the panel is 20 ft wide by 12 ft high, about two-thirds of the total wind pressure is concentrated at the floor levels by the spandrel beams, and, if the walls do not fit tightly against the columns, practically all the wind pressure may be thus concentrated. Hence, the writer feels that the moments of Column (5), Table 11, represent the most probable set of values. Mr. Wilbur seems to concur in this opinion.

Mr. Letherbury studied an interesting step-back frame by the simplified method and by the Bowman method. Despite the set-back, this is an extremely regular frame, which accounts for the small error (20%) shown by the statically determinate (Bowman) method. No such agreement could be expected where the variation of K-values corresponds to a normally irregu-

lar frame. Mr. Letherbury makes a valuable suggestion in regard to the use of the writer's method of estimating deflections for the study of tower deflections and vibrations.

Mr. Weiss emphasizes the time consumed by the wind-stress analysis of a tall building. He observes that the 56-story Irving Trust Company Building

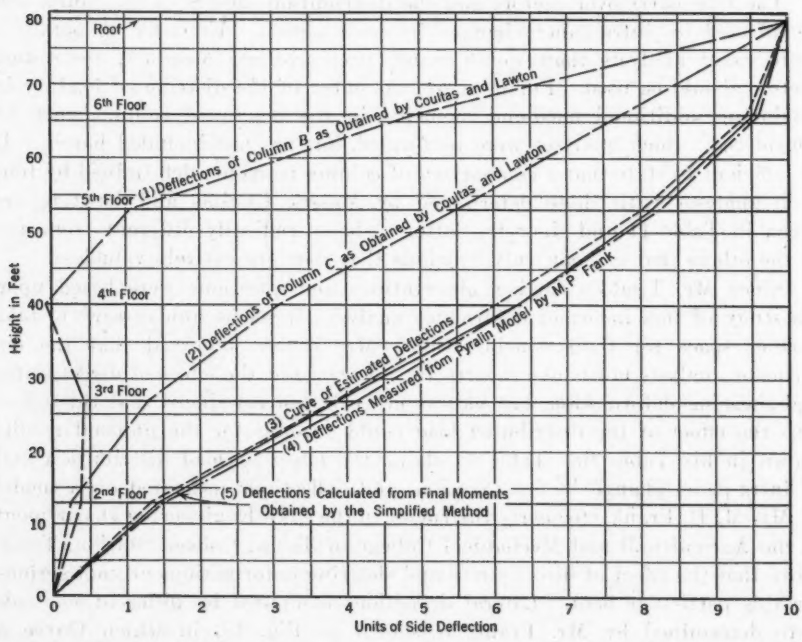

FIG. 18.—COMPARISON OF LATERAL DEFLECTIONS FOR COULTAS AND LAWTON BENT.

contains 6 200 wind-resisting members in its entire frame. The writer counts about 620 members in a single bent, as shown in Table 5, and, hence, he concludes that there must be about ten such bents. In a very regular building all ten such bents may be practically identical, but it will be assumed that there are ten variations to be analyzed separately. A wind-stress analysis of a tall frame can be performed by the simplified method in considerably less than 10 min per member. The time consumed would be about 500 hr if the frames were so irregular from top to bottom that a statically determinate analysis could not be used over any part of the frame. Is it unreasonable to detail five men for two or three weeks to obtain such information? Can not they be allowed two or three months, if necessary, in which to produce a frame truly proportioned to resist wind stresses in an economical manner? If the structural frame is irregular, is not the engineer justified in spending 0.1% of the cost of a skyscraper in really solving the wind-stress problem?

Mr. Weiss feels that the writer has "overlooked one of the peculiar characteristics of modern tall buildings." He refers to the use of stiff columns and slender girders in the lower stories. On the contrary, the simplified method

was designed to apply to all such cases. Fig. 13 is purported to show that the task of applying the simplified method to a structure such as the Irving Trust Company Building would become an impossible undertaking because of slowness of convergence of moments. The writer believes that Mr. Weiss confused the simplified method and the general method of successive corrections, as adopted by Sub-Committee No. 31, Committee on Steel, of the Construction Division, on Wind-Bracing in Steel Buildings, because his criticism would be applicable to the latter, but not to the former. To show that the application of the simplified method to a frame with stiff columns and slender girders is not "such a Herculean undertaking as to overshadow

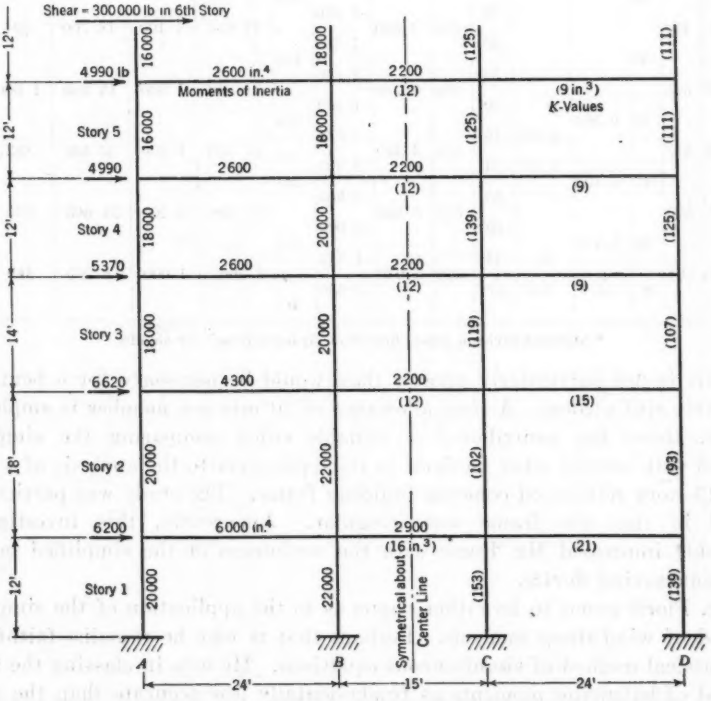

FIG. 19.—PROPERTIES OF FRAME HAVING STIFF COLUMNS AND FLEXIBLE GIRDERS.

completely the task performed by Professors Wilson and Maney in solving sixty simultaneous equations," the writer has analyzed the frame shown in Fig. 19. This frame maintains the same essential features of stiff columns and slender girders found in the Irving Trust Company Building. Deflections are estimated in Table 12 and moments are balanced in Fig. 20. An interesting characteristic is the development of negative moments at the top of the lower-story columns, which, incidentally, necessitated the introduction of correction moments into Story 1. However, the labor involved in the

TABLE 12.—COMPUTATIONS FOR ESTIMATING STORY DEFLECTIONS FOR FRAME HAVING STIFF COLUMNS AND FLEXIBLE GIRDERS. PROPERTIES GIVEN IN FIG. 18.

Story	Height, in feet	ΣK_c, in inches³	ΣK_g, in inches³	r	n	Moment division, percentage	Shear, in kips	Total moment, M_t, in kip-feet	Total column moments, in kip-feet	$E\theta \div 1\,000$	Deflections, in Inches: $E\theta_{avg.} \times h \div 1\,000$	Deflections, in Inches: $E\delta \div 1\,000$	Deflections, in Inches: $E\Delta \div 1\,000$	Fixed-End Column Moments, in Inch-Pounds ÷ 100 000: A or D	Fixed-End Column Moments, in Inch-Pounds ÷ 100 000: B or C
6	12	472				50	300	3 600							
						50			1 800	121					
			30												
5	12	472				50	305	3 660	1 830		17 600	1 100	18 700	865	974
						50			1 830	123					
			30												
4	12	528				50	310	3 720	1 860		18 200	1 000	19 200	1 000	1 110
						50			1 860	130					
			30	0.061											
3	14	452			0.222	46	316	4 420	2 030		21 500	1 600	23 100	883	982
					0.261	54			2 390	126					
			42	0.100											
2	18	390				50	322	5 800	2 900		21 400	3 200	24 600	635	698
						50			2 900	72					
			58	0.119											
1	12	584			∞	33*	328	3 940	1 310		5 200	1 000	6 200	359	396
			∞	∞	2 ∞	67*			2 630	0					

* Moment division where first floor girder is relatively slender.

analysis is not particularly greater than would be necessary for a bent with relatively stiff girders. A time allowance of 10 min per member is ample.

Mr. Reese has contributed a valuable study comparing the simplified method with several other methods in its application to the analysis of a practical 15-story reinforced concrete building frame. The study was particularly useful in that the frame was irregular. Apparently, this investigation favorably impressed Mr. Reese with the usefulness of the simplified method as a time-saving device.

Mr. Floris seems to be rather vague as to the application of the simplified method of wind-stress analysis. Perhaps that is why he remains faithful to the classical method of simultaneous equations. He errs in classing the Cross method of balancing moments as fundamentally less accurate than the slope-deflection method, and he is further mistaken in assuming that the simplified method is tedious in practical cases, as has been shown by several discussers. Contrary to Mr. Floris' viewpoint, the writer feels that the importance of the elementary arithmetical procedure of the simplified method needs further emphasis.

Mr. Gedo is correct in emphasizing the importance of direct stress deformations on the moments in a Vierendeel truss, although the writer made no error in stating that direct stress deformations usually are neglected. The writer made an exhaustive study of this problem several years ago[35] and

[35] Thesis presented to the Univ. of Illinois in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering.

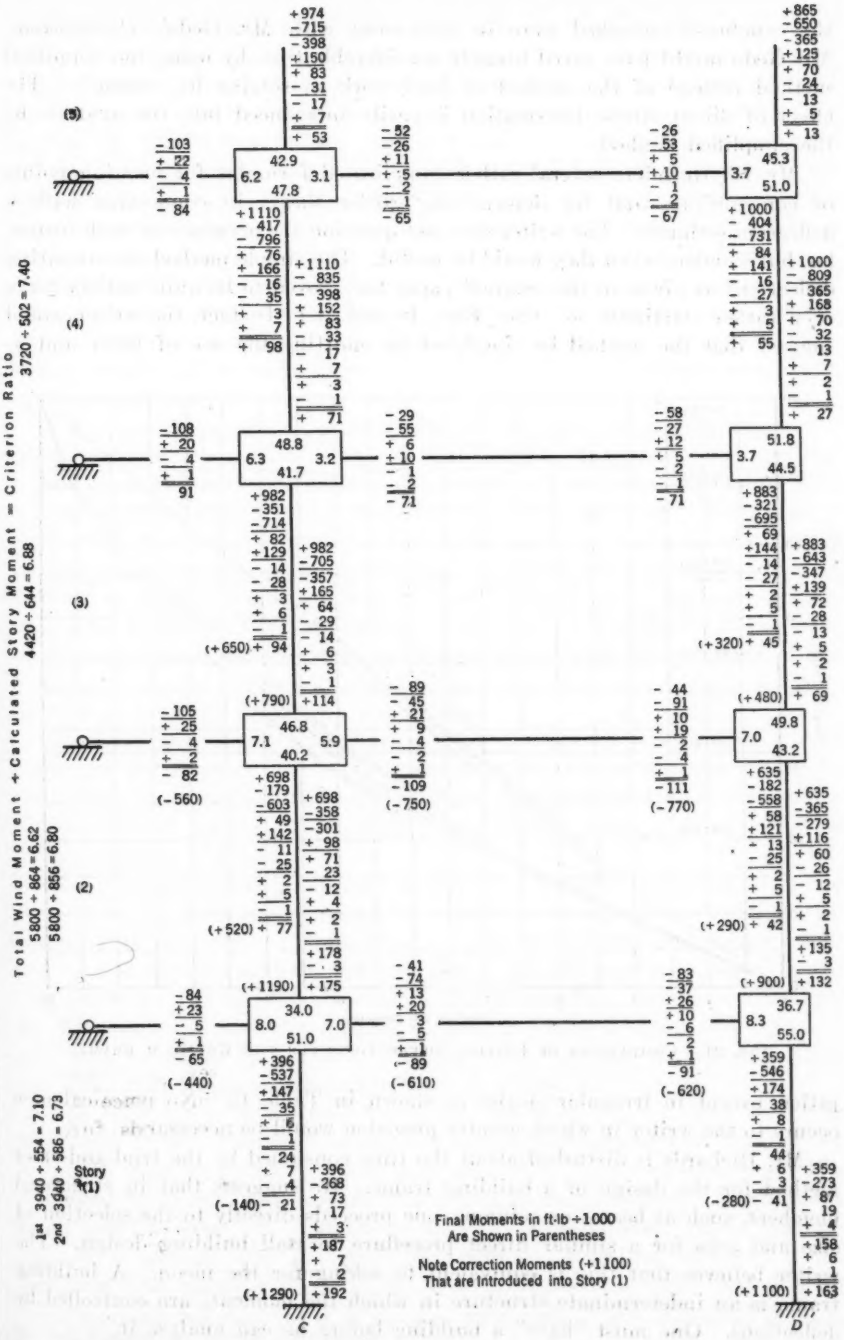

FIG. 20.—ANALYSIS BY THE SIMPLIFIED METHOD OF A FRAME HAVING STIFF COLUMNS AND FLEXIBLE GIRDERS. PROPERTIES GIVEN IN FIG. 18, FIXED-END MOMENTS FROM TABLE 12.

the conclusions reached were in agreement with Mr. Gedo's observations. Mr. Gedo might have saved himself considerable time by using the simplified method instead of the method of least work in solving his example. The effect of direct stress deformation is easily introduced into the analysis by the simplified method.

Mr. Skytte offers several rather complicated formulas for locating points of contraflexure and for determining girder shears in connection with a deflection estimate. The writer does not question the accuracy of such formulas, but wonders when they would be useful. The simple method of estimating deflections as given in the original paper has proved to be quite satisfactory; even rather startingly so. (See Figs. 18 and 21.) In fact, the writer would suggest that the method be simplified by omitting the use of the r and n-

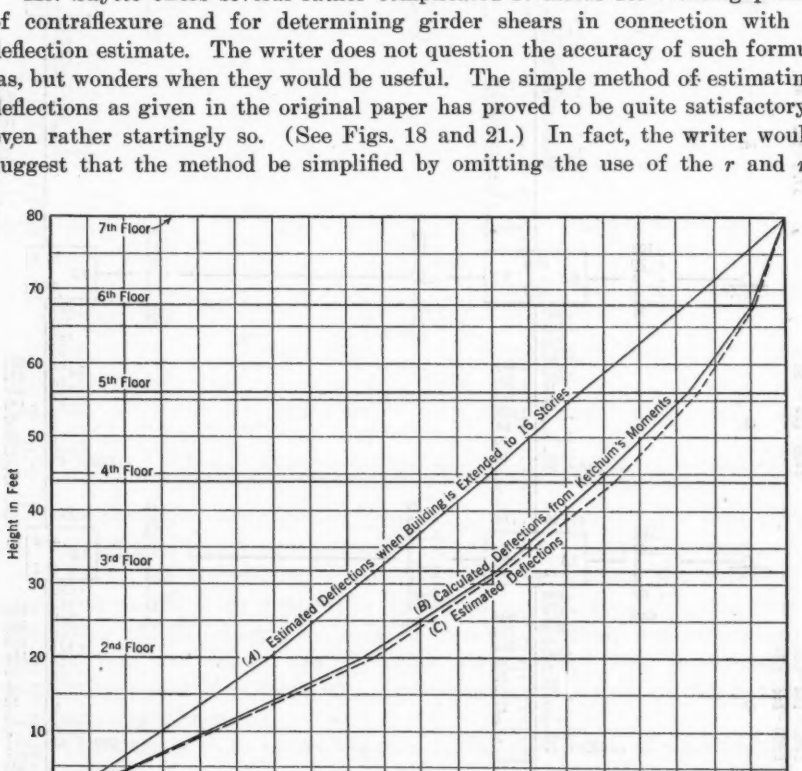

FIG. 21.—COMPARISON OF LATERAL DEFLECTIONS FOR THE KETCHUM FRAME.

ratios, except in irregular stories as shown in Table 12. No practical case occurs to the writer in which greater precision would be neecssary.

Mr. Richards is disturbed about the time consumed by the trial-and-error method for the design of a building frame. He suggests that in structural members, such as beams or columns, one proceeds directly to the selection of size and asks for a similar direct procedure for tall building design. The writer believes that this is equivalent to asking for the moon. A building frame is an indeterminate structure in which the moments are controlled by deflections. One must "have" a building before he can analyze it.

There is a possible answer to Mr. Richard's question; namely, a building might be proportioned to justify the assumptions of a statically determinate method of analysis; then the true wind moments would be as assumed and the design would be completed in a single operation. The only difficulty here is that "the tail is wagging the dog." A method of analysis is set up on a pedestal, and the more important procedure of design is relegated to the background. The designer can study economical proportions only through a limited range and a truly economical structure is seldom possible. It is surely wasteful always to force points of contraflexure to the centers of members, and to attempt to suggest to the structure which members shall carry direct stress, approaches the impossible unless the frame is very regular. There seems to be a popular fetishism to this ideal of design, but the writer doubts that it will survive or even that it can be strictly applied to an irregular frame.

Like most extremely practical men, Mr. Spurr has so frequently encountered difficulties produced by structural details that he may perhaps tend to under-emphasize the usefulness of a theoretical tool of analysis just as a college professor may tend to over-emphasize its importance. When one is faced with the daily realization that angles, split beams, and knee-braces must be used more or less indiscriminately, and that all are commonly assumed to produce rigid connections when the structure is analyzed (which, of course, is not true) the assumption of points of contraflexure at the mid-height of columns takes on a less serious aspect. However, despite these disturbing factors, the writer prefers to place his faith in a method of analysis that takes elastic deflections into account.

Mr. Spurr errs in stating that the author has not established a method of caring for the effect of column deformations. The matter receives full attention under the heading, "Effect of Column Deformation." The evaluation of the effect of column deformation is not difficult when a second analysis follows revision of proportions.

Mr. Fleming supports, at least partly, the writer's contention that the method of successive corrections, of which the simplified method is one variation, is "feasible" for application to practical tall building design where the slope-deflection method and the other classical methods were beyond reasonable use. The writer considers this particularly significant, because Mr. Fleming's statically determinate methods are those that careless designers are likely to try to use on irregular structures, whereas Mr. Fleming himself states that irregular structures "should receive special study."

The six-story bent analyzed by M. S. Ketchum, M. Am. Soc. C. E., is mentioned by Mr. Fleming[25] as showing a variation of criterion ratios from 5 to 50% when analyzed by the simple method of balancing K-values. His statement is correct, but perhaps misleading to some readers. No different result could be expected. The balancing of K-values is based on the assumption of a straight-line (or gradually curved) deflection diagram. The Ketchum bent shows a variation of 20 to 1 between the total moments in

[25] "Steel Mill Buildings," by M. S. Ketchum, Fifth Edition, 1932, Chapter on "Stresses in Stiff Frames by Moment Distribution."

the first and sixth stories, while the K-values of the columns are scarcely doubled and the girders are unchanged. A tall building, on the other hand, shows a corresponding increase in K-values and story moments from top to bottom. The tall building will commonly approach a straight-line deflection curve while that for the Ketchum bent will be roughly parabolic.

The curves of Fig. 21 are deflection diagrams for the Ketchum bent. The two similar curves (B and C) represent the deflection of the six-story bent as estimated by the writer's method and as calculated from the Ketchum moments. The agreement between these curves shows that the moments obtained by the simplified method (based on the estimated deflections) would check the Ketchum moments closely. The third curve (A) approaches a straight line and is the estimated deflection diagram for the lower six stories of Professor Ketchum's bent if it is extended upward to form a 16-story building. Note that for this practical case the deflection curve does approach a straight line and the moments could be obtained within a range of error of 10% by following the procedure of balancing K-values, as illustrated for the American Insurance Union Building, in Fig. 5. Correction moments would be required in the lower story only. Incidentally, it seems that the idea of introducing correction moments made little impression on the discussers. Experience will show that this is one of the most valuable tools of the simplified method.

It seems significant that members of the profession, most of whom are practicing engineers, are sufficiently interested in the problem of wind stresses to submit discussions. If such interest can be stimulated, it will most certainly lead to improvement in present methods of tall building design.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESSES BY SLOPE DEFLECTION AND CONVERGING APPROXIMATIONS

Discussion

BY MESSRS. S. S. GORMAN, AND A. FLORIS

S. S. GORMAN,[11] ASSOC. M. AM. SOC. C. E. (by letter)[11a].—The author states that in wind load distribution "it is usual to assume that the frame carries the entire load." This may be entirely unjustified by the conditions in many instances. In fact, in some cases on which the writer was associated, the walls were so rigid as to carry more than 90% of the lateral loading leaving very slight amounts for the individual bents in the frame. This is a condition which many engineers are prone to ignore, and results in needless waste of material and labor. While it may be considered as an additional factor of safety to carry the entire loading on the frame, this is a waste of the client's money, for which he derives no material benefit. Furthermore, is one justified in using a refined, accurate method of analysis when the frame loading is so small a percentage of the whole, or when the distribution to the individual bents is uncertain? There are numerous occasions when a so-called accurate method is warranted, but the designing engineer must use good judgment in deciding when.

Mr. Goldberg has referred to a "composite bent" in treating the distribution of loading among the parts of the frame. This is an extremely delicate and important process, and, in the writer's opinion, has been too lightly touched upon. The frame of a building, consisting of a number of individual bents, is often unsymmetrical about the center line of the building; that is, the individual bents are dissimilar, or the walls are rarely of equal rigidities. How often are the parallel walls on opposite sides alike? These conditions result in the center of gravity of the loading not coinciding with the center of gravity of the resisting elements, thereby tending to twist the structure about a vertical axis. This torsion should be provided for but often it is not. A very common case is the building on a corner lot, built adjacent to

NOTE.—The paper by John E. Goldberg, Jun. Am. Soc. C. E., was published in May, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1933, by Messrs. G. A. Maney and L. E. Grinter.

[11] Cons. Structural Engr., San Francisco, Calif.

[11a] Received by the Secretary December 2, 1933.

other buildings, with very rigid rear walls without openings therein and with the street-front walls containing numerous openings.

The California earthquake of 1906 made apparent to Western engineers the necessity for more consideration and accuracy than had hitherto existed for determining and providing for lateral deflections, distortions, and stresses in the design of structures. While the fundamental principles for such determinations were understood implicitly, the labor and time required in making the necessary computations by the methods of analyses which were then available, made them impracticable. Many engineers devoted a great amount of time and study in an effort to simplify such computations so that they would be rapid and accurate, but generally without practical success. The contributions of Messrs. Hardy Cross, Wilson and Maney, Nishkian, Grinter, Morris, Goldberg, and many others, that have appeared in recent publications have a value to the structural engineer greater than is generally understood by the profession.

In California the necessity of providing for lateral forces is accentuated by seismic conditions, and shocks which have occurred subsequent to that of 1906 have emphasized this fact. They have been the cause of great increases in earthquake insurance rates on structures and have prompted financial interests which loan money for building purposes to demand earthquake insurance as a part of their consideration for loans. Several of the latter have withdrawn such demands where it could be shown that proper and competent attention had been given to lateral forces in the design and the building had been constructed under adequate inspection and supervision.

Mr. Goldberg's paper seems to be another adaptation of the slope-deflection method very similar to that developed in July, 1930, in the Bureau of Building Inspection, City of San Francisco, by S. M. Cotten, M. Am. Soc. C. E., under the direction of John B. Leonard, M. Am. Soc. C. E.[12] These papers are as accurate as the slope-deflection method and produce identical results therewith when the converging angular rotations are balanced a sufficient number of times. Of course, the author's paper is subject to the same primary, fundamental assumptions as the slope-deflection method, and its accuracy must be gauged by the correctness of these assumptions. Mr. Goldberg's paper is similar in principle to the method of Hardy Cross, M. Am. Soc. C. E., entitled "Analysis of Continuous Frames by Distributing Fixed-End Moments,"[13] except that instead of balancing moments it balances the angles of rotation of the joints of a frame and the deflections thereof, computing the final moments after these are balanced.

This paper may be equally adaptable to vertical or horizontal loading, to symmetrical or unsymmetrical loading or framing, to individual bents or to any number of bents comprising a structure, and to rotation of or translation of the joints of the bent. The simplicity, rapidity, and accuracy which may be obtained in analyzing a frame is indeed surprising when one becomes adept with these operations. In an individual bent one must determine an

[12] *Western Construction News*, February 10, 1931.
[13] *Transactions*, Am. Soc C. E., Vol. 96 (1932), p. 1.

initial set of values of the angles of rotation of the joints and their corresponding deflections. Balancing is then commenced and the number of repetitions required is dependent upon the accuracy obtained in the initial assumed angles and deflections. One can readily note that a poor selection of these initial values will result in a waste of time; so it behooves one to start with a value as close to the final value as it is possible to predetermine.

The loads carried by any one bent may be apportioned so that the part of the total load at any floor line of the structure will be in inverse proportion to the deflections at that point in the several bents and walls, if each of these bents is considered independent and identically loaded. This is based upon the assumption that the floor acts as a rigid diaphragm. It is obvious that unless the relative rigidity of the several bents and walls is constant for their entire height the ratio of the deflections will vary from floor to floor, and the part of the total load at any floor line, carried by each of the several bents and walls, will also be a variable from floor to floor.

In carrying through the balancing of the parts of the frame, only the relative values of rigidities, angles, and deflections are material; hence for brevity and expediency one may eliminate all constants, if one so desires, from his computations until the parts are finally balanced. These constants are then utilized in obtaining final moments.

One of the advantages of this method is that, within reasonable limits, errors in computations may creep in which will not prevent obtaining the correct solution, but will merely postpone its ultimate attainment. Furthermore, if a change in an angle or a member is necessary, a study will show that the influence in most cases diminishes very rapidly as the angles considered become more remote from each other. One soon learns in such a case, that it is sufficient to consider only that part of the structure and loads which have a material influence on the angle or group considered.

Another advantage is that the angles and deflections corresponding to the several elements of a combined loading may be combined algebraically, and that the moments derived from this summation will be those due to the combined loading. In treating the frame for vertical loading only, initial values of all the angles may be assumed as zero and the balancing proceeded with, obtaining results without excessive expenditures of time.

A great time-saver was presented in the method of the San Francisco Bureau, previously mentioned, which would improve the method presented by Mr. Goldberg very materially, namely, an "equivalent symmetrical bent" of one span for use while analyzing any number of spans in an irregular bent. This single-span symmetrical bent is considered to represent temporarily, the actual bent as to number of stories, story heights, and total distributed rigidities. Due to its symmetry only one column and only one joint at each floor line need be considered. The ΣK of the columns of the equivalent bent equals the ΣK of the columns of the actual bent at any story, and the K-value of the girder of the equivalent bent equals the ΣK of the girders of the actual bent at any floor level. The purpose of this equivalent bent is to determine approximately, and very simply, certain required proper-

ties of the actual bent, having which the analysis of the latter is greatly expedited. This consists briefly in balancing the angles of the equivalent bent, obtaining the moments in the ends of the girders and, with these, calculating the deflection in each story. These moments, deflections, etc., are then transferred to the actual bent and the balancing of the actual angles is commenced. By successive approximations any degree of accuracy can be obtained by carrying the balancing to the extent desired.

During the years 1930 to 1933, inclusive, the method of the San Francisco Bureau has been used by a number of engineers in the making and checking of actual designs of structures, while the San Francisco Bureau has made wide use thereof in the checking of plans submitted for building permits. Due to the parallelism of the two methods, these facts would indicate the practical value of Mr. Goldberg's paper in providing a simple, rapid, and accurate method of analysis for the practicing structural engineer.

While the method of analysis of the rigid structural frame by converging angular approximations apparently has no theoretical limitations, it should not be advocated that it be applied to all possible problems of the rigid frame. Most of the accepted methods of analysis, whether analytical or graphical, have some particular field of usefulness in which they excel, and the choice of method best adapted to the particular problem at hand is one of the major functions of the structural engineer.

Progress is only attained through repeated attempts to better the tools available for one's work. The present paper and many others crystallize the thought of the profession on advancement in the art of rigid frame analysis—and the World must progress!

A. FLORIS,[14] ESQ. (by letter)[14a]. — In this clearly written paper the author analyzes the wind stresses in multiple-story frames by means of formulas based on the slope deflection method.

The difficulties that arise in analyzing statically indeterminate structures are due mainly to the necessity of solving the simultaneous elastic equations by the tedious process of successive elimination of the unknowns. From the practical point of view, of course, this is a serious handicap.

In order to overcome this difficulty methods have been developed which are comparatively simple. Instead of using approximate formulas which by repeated corrections can be made to approach the exact values, as the author does in his analysis, it is more convenient to arrange the exact slope deflection equations in such a way that they can be solved quickly by the principle of iteration (see, for example, the excellent method introduced by Professor G. Unold in which the story equations are combined with the joint equations of the frame so that their number is reduced considerably[15]).

In spite of these more efficient methods the author's analysis is interesting and instructive. It deserves the careful study of those interested in the stress analysis of tall buildings.

---

[14] Civ. Engr., Los Angeles, Calif.

[14a] Received by the Secretary November 6, 1933.

[15] "Die praktische Berechnung der Stahlskelettrahmen," von Georg Unold, Berlin, 1933.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EARTHS AND FOUNDATIONS PROGRESS REPORT OF SPECIAL COMMITTEE

Discussion

BY FREDERIC R. HARRIS, M. AM. SOC. C. E.

FREDERIC R. HARRIS,[70] M. AM. SOC. C. E. (by letter)[70a].—It is highly encouraging to engineers to note that progress is being made in the complicated subject of earths and foundations. These problems have been attacked by some of the ablest minds of the Engineering Profession, but on the whole the progress has been unsatisfactory as compared to engineering advances in the design and construction of more determinate structures.

Several factors contribute to this situation, among them the following: (1) Lack of publicity of the experiences encountered; (2) lack of appreciation of the importance of the problem among engineers, architects, and owners; (3) insufficient explanatory information; (4) bearing values assigned by various building codes; and (5) the old method an easy and direct one.

(*1*) *Lack of Publicity of Experiences Encountered.*—As pointed out in the report this is a serious situation, and the reluctance to release information is not chargeable to engineers so much as it is to owners and others.

Unfortunately, engineers are retained by owners or by agents of the owners, such as architects, and frequently foundation difficulties and the problems involved only come to the engineer's attention when the danger signals are flying and when settlements, cracks, and distortions threaten calamity. Then, of course, the owner and the architect are not anxious (even though the trouble is corrected and the structure saved) to advertise their troubles or to expose mistakes. One of these occurred about 1926; the structure, a large one involving several million dollars, before completion, started settle-

NOTE.—The Progress Report of the Special Committee on Earths and Foundations was presented at the Annual Meeting, New York, N. Y., January 18, 1933, and published in May, 1933, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: August, 1933, by Messrs. L. C. Wilcoxen, H. de B. Parsons, William P. Kimball, and T. A. Middlebrooks; September, 1933, by Messrs. Daniel E. Moran, and A. E. Cummings; October, 1933, by Messrs. Edwin J. Beugler, Jacob Feld, George D. Camp, and Charles Terzaghi; November, 1933, by Messrs. John H. Griffith, Harry E. Sawtell, and O. K. Froehlich; and December, 1933, by N. J. Durant, Assoc. M. Am. Soc. C. E.

[70] Rear-Admiral, U. S. N. (*Retired*); Cons. Engr. (Frederic R. Harris, Inc.), New York, N. Y.

[70a] Received by the Secretary November 14, 1933.

ments, which finally exceeded 1 in. per month at the center of the foundation. It was only part of a general plan of improvement of wide extent and its failure would have been a calamity to important interests, and to the name and standing of a prominent firm of architects.

Hasty subsoil examinations were made, which revealed thick blankets of deep strata of clays and mud. The structure was saved by an unusual and heretofore untried method of pre-compression and a reduction of the liquid contents (voids-ratio) of the yielding soils. A year or so after the satisfactory completion of this work, when the structure was in use, the writer wished to inform the Engineering Profession of this experience. The owners finally consented to such publication, but the architects objected. The writer has had several such experiences.

(*2*) *Lack of Appreciation of Importance of the Problem.*—In some cases, engineers and architects (at considerable time and expense), will use great refinements in the computation of the stresses in the superstructure, and will go to considerable trouble in studies and investigation in order to secure an economical design; then they will endanger the entire project by locating it on improper foundations.

(*3*) *Insufficient Exploratory Information.*—It is well known that in many cases, little attention is given to this phase of the work. Ignorance, pressure for speed, comparison with structures not similar to the one on hand, and unwillingness on the part of the owners or architects to spend the necessary money, are all factors that prevent the securing of proper data on which to form a judgment. As an illustration, in the special example cited under Factor (1), the hurried and uncertain work required to save the structure cost more than $250 000. Proper preliminary exploratory work with necessary studies, would have cost less than one-tenth of this.

(*4*) *Bearing Values Assigned by Various Building Codes.*—This situation is one that tends to lull some engineers into a sense of security so that they take chances they would avoid except for these rulings.

(*5*) *Old Method Easy and Direct.*—Naturally, it is much to ask even the Engineering Profession to abandon an easy method and to substitute in its place a more complicated and expensive (first cost) system of analysis, especially when the latter is at times not clearly understood or when it is not subject to exact determination.

In spite of the unscientific procedure followed, and because in most cases the results have not been disastrous, the problem has not received the attention it deserves. Therefore, it is gratifying to note that it is being studied so thoroughly and treated on a scientific basis, and especially that it appears to be enlisting such general engineering interest. During the past few years public expression has been given to ideas that have been in the minds of some engineers and who have actually applied them in practice, but who have not found time or opportunity, or have been otherwise hampered in giving the necessary publicity to their ideas and experiences. Such opportunities are now being offered through the work of Professor Terzaghi, the Special Committee on Earths and Foundations, and others.

Before Boussinesq's formula was given the recognition which through Professor Terzaghi's work it now enjoys, the writer used this system of superimposition to express the greater intensity of pressure induced immediately under the load, and the angle of spread used was 60° with the horizontal.[77] This angle suggested itself as a result of a series of tests conducted in 1912 at the Brooklyn Navy Yard in connection with the construction of Dry Dock No. 4. In these tests layers of different colored sands were placed in a box with one of the sides of glass; model footings made of wood were loaded, and the dishing action resulting in the layers was noted. Furthermore, the line of fracture of the soil under the footing (approximately 60° to the horizontal and beginning at the edge of the footings) was observed.

In these same tests, the model footings were pulled up to note the effect of an uplifting force on the structure and the extent to which the footing affected the soil surrounding it. Although not conclusive, nevertheless evidence exists of certain similarity between the actual results and what is expected of Boussinesq's formula. The sand used in the experiment was of a uniform character, and no attempt was made to determine the effect of variations in the character of the material.

These tests were made for purposes of ready visual elucidation of load distribution, but at the same time tests were also made with models to a scale of 1:10, settlement, uplift, and other features being recorded. In this connection, it would be well if the Committee, when applying Boussinesq's formula to a practical problem, would explain the maximum angle, ϕ (Fig. 2), used in obtaining increments of p_z on a given point from various adjacent loads, P. It is believed that, for practical purposes, 30° will serve the purpose, and, in fact, has been used by the writer on a recent problem.

The work done by various investigators on the nature of clay and its behavior under load also indicates a distinct step forward, even if the first impressions may have to be altered in the light of further development. The importance of ascertaining soil characteristics impressed the writer many years ago, as he and so many engineers at that time were dissatisfied with the information available from wash borings. It was very apparent then that it was highly desirable to secure soil samples as nearly as possible in their natural state and arrangement, so that these might be, not only subjected to inspection, but also that they might be available for testing.

While investigating the failure of the Pearl Harbor (Hawaiian Islands) Dry Dock in 1913, the writer attempted to secure subsoil samples by core borings. Although the only available rig was not entirely successful, cores were obtained by driving a pipe ahead of the casing. These samples were analyzed, examined, and photographed under a microscope, and their specific gravity and water content were obtained.

In 1916, in preparation for the design and construction of dry docks in Philadelphia, Pa., and Norfolk, Va., the writer made a contract for exploratory borings. A special core barrel was designed to secure undisturbed

[77] *Engineering News-Record*, November 24, 1932.

samples, but this tool was not completely successful and final resort was made again to driven pipes.

Recognizing the importance of ascertaining the rate of compressibility of soils, such as clays, mud, and silt, the writer in 1926, secured samples of sub-soil beneath the large structure referred to under Factor (1), by means of a 4-in. coring apparatus. Samples were examined carefully, including compression tests in the laboratories of the University of Hawaii. From the resultant data an attempt was made to ascertain the probable total future settlement of the structure, as well as the extent to which the subsoil had to be pre-compressed (required in the method used to save the structure). At that time neither the facilities nor the theories were well enough advanced and developed to classify the material in the modern way; nevertheless, an approximate value was determined.

In determining the consolidation of soils, the writer (as well as other engineers) has held the theory that soil particles, no matter how fine, are separated from each other by films of the ground-water in which they are immersed, or by water and finer soil in colloidal state; that on account of the natural over-burden of soils above them, and artificial weights, such as buildings (or for both reasons), a pressure or load is brought on the particles that tends to press the films out. This consolidation to the point of mechanical contact of soil particles is resisted by the capillary tension of the fluid film, to the point where the pressure is in excess of such resistance. When flow starts and consolidation begins, the flow is resisted, of course, by the fluid friction. A great part of this resistance is also that required in overcoming the capillary tension of the fluid between surrounding and more remote soil particles, because the water must find its way either to come to the surface or to raise the ground-water level. Very rapid and continued vibration has a definite effect on the resistance due to the capillary tension. A box filled with soil, saturated with water (but where the water does not appear on the surface), if subjected to continual jarring or vibration, will soon show evidence of surface saturation, and, finally, water appears on the surface; or, a more familiar illustration is: A truck, loaded with very fine sand and some clay, which at first looks dry, but on arrival—after the jarring and vibration of trucking some distance—has turned its contents into what is familiarly known as "bull's liver."

In the remoulding of clays, the change that takes place could be explained by the breaking up of the equilibrium in the capillary tension because of the re-arrangement of the particles from the position into which they had been forced by the pressure of the over-burden. This produces another re-arrangement of the forces holding the particles apart and inasmuch as this re-arrangement has not taken place without confining the particles, the films may force them apart again so that when pressure is once more applied under load, they will again be forced closer together, producing a reduction in volume and, consequently, consolidation.

In view of the progress made in tamping concrete with vibrators it has occurred to the writer that perhaps an index of the nature of the clay and

its compressibility may be obtained by vibrating the sample and finding its period of vibration, all with the view of finding a possible relation between the period of vibration of the sample and its compressibility.

The settlement in alluvial or other soils observed in some earthquake countries[78] perhaps may be ascribed to the continued and rapid vibration produced by an earthquake disturbance of the soil which would tend to disrupt the equilibrium incident to the capillary tension. Of course, no relation may exist between the period of vibration and the consolidation of the soil, but it is hoped that the suggestion may appeal to experimenters or others in position to investigate such a relation and that a few experiments may be tried to determine the merits of the question. The investigations on both pressure distribution and the behavior of clay under load are in the stage of development and, in order to progress, it is essential that the theories be applied and the results noted.

Recently, the writer encountered a soil which borings disclosed to be rather complex and offering the possibilities for several types of foundations that could be used. The ordinary wash borings with dry sample core disclosed two layers of clay which were variously interpreted as soft, plastic, hard, and compact. The conditions appeared ideal, however, for the application of the latest theories on soil mechanics to determine the most suitable type of foundation. Consequently, three 6-in. core borings were made, securing undisturbed samples of clay from an analysis of which compressibility curves were obtained.

The intensities of pressure on the various strata of clay were computed with the aid of Boussinesq's formula and lines of equal intensities of pressure for different types of foundations drawn. In addition, the probable total settlement and the settlement after the period of construction were determined with the aid of the compressibility curves, and lines of equal probable settlement were drawn. On the basis of these studies, a type of foundation was decided upon which would not have been considered adequate, or, at best, would have been considered hazardous without the investigations, and which affected a saving in the structure of about $100 000, besides having other important advantages.

This is the first case, to the writer's knowledge, in which the modern theory of soil mechanics has been applied to determine the proper and most economical type of foundation, and which has resulted in a re-arrangement of the superstructure to provide for probable settlements.

---

[78] *Proceedings*, Am. Soc. C. E., May, 1932, Pt. 2.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABILITY OF STRAIGHT CONCRETE GRAVITY DAMS

Discussion

BY MESSRS. WILLIAM P. CREAGER, F. W. HANNA, LARS R. JORGENSEN, AND I. M. NELIDOV

WILLIAM P. CREAGER,[21] M. AM. SOC. C. E. (by letter)[21a].—This ably presented paper deals with a subject that is very important to the designers of high, solid, gravity dams. Included in Table 8 is a mathematical demonstration of the fact, frequently expounded by the writer,[22] that the factor of safety against sliding is much less for high dams than for low ones.

This feature is more impressive when shown diagrammatically. In Fig. 9, the writer has plotted the relation of friction and shear to the factor of

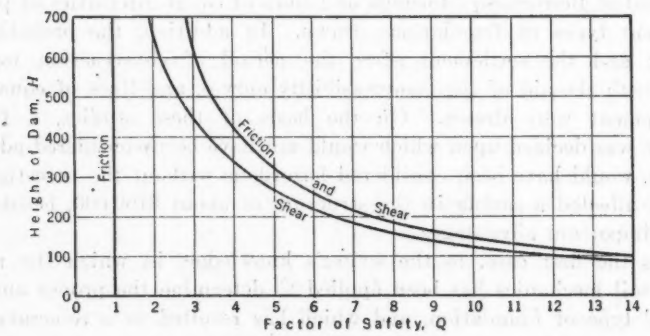

FIG. 9.—EFFECT OF HEIGHT OF DAM ON FACTOR OF SAFETY AGAINST SLIDING

safety as applied to Column (3) in Table 8. The decreasing effect of shear for high dams is very clear; it contributes relatively seven times as much to the safety of a 100-ft dam as to that of a 700-ft dam.

NOTE.—The paper by D. C. Henny, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by H. de B. Parsons, M. Am. Soc. C. E.; and December, 1933, by Messrs. A. A. Eremin and Calvin V. Davis.

[21] Chf. Engr., The Power Corporation of New York, Buffalo, N. Y.

[21a] Received by the Secretary November 15, 1933.

[22] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 197.

The factors of safety, Q, in Table 8 are tentative pending "the results of additional research." In the writer's opinion they are too great unless certain precautions are taken. Mr. Henny finds that experiments indicate a value of 800 lb per sq in. for shear. This he has reduced tentatively to 400 lb per sq in. on account of uneven shear distribution which he uses in his example. The value of 400 lb per sq in. may be reasonable for shear in solid concrete, but it is not applicable to joints.

It is obvious that shrinkage of new concrete when placed on old concrete or rock foundations will impair, greatly, the adhesion and shearing strength. As an example, Joseph Wright, M. Am. Soc. C. E.,[23] found that only 4% of the area of the base of a large pier had bonded with the rock surface on which it was carefully placed.

Thus, it is obvious that shear cannot be relied upon unless all joints, including those at the rock surface, are properly keyed and the sectional area of keys alone are considered as resisting shear. With the most efficient key system, the effective area of shear at any joint would be 50% of the joint area. Applying the author's 400 lb per sq in. on 50% effective area of joint results in the use of 200 lb per sq in. over the entire area. This adjustment reduces, greatly, the factors of safety, Q, in Table 8, resulting in $Q = 1.02$ for a 700-ft dam, as compared with 4.24 and 7.51 for a 200 and a 100-ft dam, respectively, in the case of heel loading equal to 0.44 H.

If one could be sure of the 400 lb per sq in. adopted as the shearing value in solid concrete blocks, these factors of safety would be sufficient even for a 700-ft dam. In this case simply roughening the foundation (if not already provided by Nature), and also the joints in the concrete, should be ample for very low dams. Medium high dams up to, say, 200 ft, should be provided with exceedingly rough or keyed foundations and bonding stones in the concrete joints. Above that height, keys in both the foundation and concrete joints should be provided. Dams of greater height than those already built lead into unexplored territory, and their design should be approached with caution, based on more adequate shear experiments.

It is interesting to note, although not the subject of this paper, that the factor of safety against overturning, to be as great as that provided in all other structures, must rely upon tension, and that the additional factor of safety provided by tension is reduced, for high dams, in much the same manner as that indicated by the author for shear.

As the criteria for the uplift force, Mr. Henny uses: (1) A unit uplift pressure evenly distributed from full head-water pressure at the heel to zero (no tail-water) pressure at the toe; and (2) an uplift force equal to the product of the unit uplift pressure and the area of voids in a given section.

The first criterion is safe enough provided care is taken to make the dam less pervious at the heel than at the toe. As for the second criterion, experiments indicate a void volume of about 10 or 12% when subjected to absorption tests. However, the void area in any section may greatly exceed the void volume. Moreover, the absorption tests would not indicate voids that would be penetrated under high pressure. Therefore, the author's use of 25 to 40%

[23] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 207.

for void area is commendable notwithstanding the inadequacy of his "small dams" tests on which it is based. The small dams might have adhered to the rock sufficiently to prevent "progressive increase of percolation or uplift pressure" even if the voids exceeded the percentage assumed.

Defects in the foundations, such as horizontal seams, are the great sources of uncertainty. In the Austin, Pa., dam, the friction of a clay-filled horizontal seam in the foundation was not adequate, but resistance was provided by the horizontal-column action of the layer of rock between the dam and the seam which extended a considerable distance down stream. When this column was cut through by the scouring overflow, the dam slid on the seam. The required resistance ("toehold") provided by the rock down stream from the dam, varies as the total water pressure or as the square of the height of the dam. In other words, the thickness of the layer between the dam and the seam, if determined by the horizontal crushing strength of rock, must be forty-nine times as great for a 700-ft dam as for a 100-ft dam; and if it is determined by the weight of the layer, it must be forty-nine times as heavy for a 700-ft dam as for a 100-ft dam.

F. W. HANNA,[24] M. AM. SOC. C. E. (by letter)[24a].—Many engineers familiar with, and experienced in, the design of dams will welcome this paper on the stability of straight concrete gravity dams even if they may not feel fully inclined to adopt the shearing strength as the principal basis for the factor of safety of such structures. The writer has found that many designers of dams consider the shearing strength on the foundation and other horizontal sections of a concrete dam as an added factor of safety against sliding, but that most of them feel constrained to use the tested wet friction coefficient as giving the minimum resistance to sliding on horizontal planes, particularly at the foundation. The ultimate shearing value used, however, is the unloaded shearing strength of the foundation or dam material, taken as 50%, or less, of the 28-day compressive strength with an appropriate factor of safety and applied uniformly from heel to toe over the horizontal section under consideration. The use of 50% is warranted by experiments made by Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E.[25]

In the ordinary gravity dam the vertical loading due to the weight of the dam and to the water pressure, and, consequently, the shearing strength, may be assumed to vary uniformly from a small amount at the heel to a maximum at the toe when the reservoir is full and *vice versa* when it is empty, with intermediate conditions of loading for intermediate stages of water levels. For this reason the minimum shearing strength should be found and used.

Equation (3) may be written (using the author's nomenclature and pressure assumptions), as follows:

$$S = S_1 + k\left(\frac{aHL}{2} - \frac{chL}{2}\right) \qquad (17)$$

[24] Chf. Engr. and Gen. Mgr., East Bay Municipal Utility Dist., Oakland, Calif.

[24a] Received by the Secretary November 29, 1933.

[25] "Tests of Concrete: I, Shear; II, Bond," *Bulletin No. 8*, Eng. Experiment Station. Univ. of Illinois, Urbana, Ill.

in which, h is the depth of water acting on the up-stream face of the dam, and c is the combined coefficient of uplift area and pressure intensity at the heel. Inspection of Equation (17) shows that the total shearing strength on a horizontal section increases as the value of h decreases and that, therefore, the total shearing resistance along any horizontal section is a minimum for full reservoir for a specific coefficient of uplift. It also shows that the total shearing strength is a minimum for a value of unity for c; that is, for a maximum uplift force. The total shearing resistance therefore, should be based on full reservoir pressure and the maximum probable value of c. For full reservoir, $h = H$, and Equation (17) may be written,

$$S = S_1 + \frac{k H L}{2} (a - c) \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

For $c = 0.5 \times 0.4 = 0.2$ and $a = 2.4$, as suggested by the author, it will be seen that the last term of Equation (18) is decreased only about 8% by the consideration of uplift in the shearing strength and, since this term is a relatively small part of the entire right member of the equation for ordinary working pressures, the effect of uplift on the total shearing strength is relatively small. This is specifically true because of the existing uncertainty as to the unloaded shearing value of concrete as determined by tests. Tests by Professor Talbot[25] show: A mean of 820 lb per sq in., with a variation from maximum to minimum of 289 lb for plain plates; a mean for recessed blocks of 932 lb, with a variation of 343 lb; a mean for reinforced recessed blocks of 1 476 lb, with a variation of 504 lb; and a mean for restrained beams of 1 167 lb, with a variation of 293 lb.

The minimum unit shearing strength in any horizontal section for reservoir full is at the heel where the net unit pressure on the foundation is nearly zero. If the weight of concrete and the uplift force and its distribution are assumed to be fully known, the theoretically perfect design of the author's triangular dam would make the foundation pressure at the heel exactly zero. In any good design it would not be much in excess of this value. In such a case the minimum unit shearing strength of the horizontal section would be at the heel for reservoir full and would be represented by the unloaded unit shearing strength of the concrete. If shearing rupture occurred at this weakest point, progressive increases would occur across the entire horizontal section and failure would result. For this reason the writer uses 250 lb per sq in.[26] for unit horizontal working shearing strength, this amount being about 25% of the ultimate unloaded shearing strength of 1:3:6 concrete.[25]

The author has accumulated data on uplift pressure that are quite helpful. His assumption of 0.5 H is commendable for reasonably good rock foundations and concrete. For gravel foundations often encountered in the construction of diversion dams the uplift pressure may approach unity, particularly where lengths of down-stream aprons rather than extensions of up-stream cut-off walls to impervious materials are relied on to reduce uplift pressure.

[26] "The Design of Dams," by Hanna and Kennedy, p. 119.

In the determination of uplift areas the writer prefers to use the following formula:

$$c' = 1.21\ v^{\frac{2}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which, c' is the area of a great circle of an assumed spherical void, and v is the volume of such a void. Expressing the relation between the volume of the assumed spherical void in terms of its radius; expressing the area of a great circle of the spherical void also in terms of its radius; and, eliminating the radius between these two equations, Equation (19) results. Although it is probable that the voids in concrete or rock have a multiplicity of shapes, they may be assumed to be spherical as a general average. It is probable also that a section of maximum voids through the concrete will be a warped surface passing through the largest number of pore spaces. However, for computation purposes the holes cut by this warped surface may be considered to be projected on to a horizontal plane. Now, it is evident from Equation (19) that, if v is taken as the proportion of the volume of the concrete occupied by voids, c' will then represent a corresponding proportion of the area of the horizontal section occupied by the area of the projected voids on it. Applying Equation (19) on the basis of this assumption to the author's uplift areas given in Line 2 of Table 8, the following values of v and of the estimated relative proportion of voids in the concrete are found:

Void area, c'	Volume of void, v	Void area, c'	Volume of void, v
0.40	0.478	0.30	0.394
0.375	0.458	0.275	0.371
0.35	0.437	0.25	0.348
0.325	0.415		

Dividing v in Line 1, by 3 and v in Line 7 by 2 to eliminate the author's safety factors used in his uplift area values, it is found that reasonable agreement with the author's percentages of voids in concrete is obtained.

In Table 6, Mr. Henny lists some assumed effective uplift pressure areas for dams founded on good rock for vertical loading at the heel, and he gives "somewhat arbitrary" effective uplift pressure areas for increased vertical loadings. For heel loadings varying from 0.4 H to H this listing indicates that the effective uplift area decreases from 40 to 25 per cent. It appeals to the writer that this is too great a decrease, and that the decrease according to Hooke's law should be that due to increased deformation of the concrete caused by increased vertical load on the walls of the individual voids. The horizontal load would consist of two parts: (1) A constant load, H, arising from water pressure on the water face of the dam; and (2) an increased load arising from the increased vertical load acting through Poisson's ratio. The increased vertical heel loading under consideration is $H - 0.4\,H = 0.6\,H$. For concrete with a modulus of elasticity of 3 000 000, the vertical shortening of the voids with diameters, d, in a unit area of horizontal section would be, $0.6\,H\,d \div 3\,000\,000$, and the lateral shortening would be about one-fourth of this amount, or $0.6\,H\,d \div 12\,000\,000 = d(H \div 20\,000\,000)$. For a head of

200 ft the change in d would be $d \div 160 = 0.00625\ d$, or a little more than 0.5% of d. This should be somewhat representative of the contraction of the pore area due to change of loading. On the other hand, the author's reduction from 40 to 25% for the change of pressure area reduces it by 37½ per cent.

The author's application of the shear safety factor to curved gravity, gravity arch, and arch dams is an interesting procedure and provides an excellent check on the general stability of dams against down-stream movement. As far as the writer knows, the idea of considering shearing resistance to rotary movement about a vertical axis in or near one end of the dam is novel. The writer has used the unloaded shearing resistance of the contact area of the entire dam as a balancing force against water pressure in all types of dams.

That the shearing resistance is a good criterion for determining the safety of well constructed dams on good sound rock foundations, with natural or artificial roughening, is feasible, is unquestionable; but its use for poor construction and poor foundations is not advisable. It is also thought that the use of the uplift force in determining the shear factor of safety is an unwarranted degree of refinement, and that the decreases in uplift area for increasing reservoir pressures are too large.

LARS R. JORGENSEN,[27] M. AM. SOC. C. E. (by letter)[27a].—Undoubtedly, the shearing strength of the concrete in a straight gravity dam may add considerably to the factor of safety of such a structure, especially if this shear is accompanied by compression in a plane perpendicular to the plane of shear. As a general rule, dams are now constructed to take advantage of all the possible shearing strength of concrete and rock by stepping the surfaces to resist a down-stream movement in the best possible manner.

When a material is subject to the ordinary compression test, there is pure compression in the direction of the axis of the testing machine and zero stress in directions normal to it. In all other directions there is both compression and shear. The shear strength and the compressive strength are interdependent. When a specimen is crushed in a compression test it is said to have failed in compression, but that does not imply that the actual failure was compression, because it may just as well have been brought about by shear, and for concrete this is generally the case.

There is fair agreement among engineers that concrete is not reliable in tension. Both the Bouzey and the Habra Dams failed above the foundation, and at the section where failure occurred a small tension was calculated. This was computed on the usual assumption of linear stress distribution and on the further assumption that there was no uplift.

The assumption of linear stress distribution on horizontal planes may be correct for the upper part of the dam, but the writer is convinced that it is incorrect near the base. This was shown quite conclusively by the English

[27] Cons. Hydro-Elec. Engr., Berkeley, Calif.
[27a] Received by the Secretary December 4, 1933.

tests,[28] and again by some simple tests carried out[29] by Donald P. Barnes, Jun. Am. Soc. C. E., so that all the experimental evidence available indicates non-linear stress distribution at the base. Mohr's assumption of parabolic shear distribution on horizontal planes[30] and Jakobsen's stress distribution according to the principle of least work,[31] both indicate the presence of considerable tension at the up-stream face of a section with about a $66\frac{2}{3}\%$ base-to-height ratio, while the usual theory shows no tension and, furthermore, this is on the assumption that the uplift is zero.

The Levy section is supposed to have a vertical pressure at every point equal to or greater than the hydrostatic pressure of the water, assuming linear stress distribution on horizontal planes. Therefore, if a seam does exist in the foundation, the water can not enter or can not spread out, because it encounters horizontal surfaces pressed together with a pressure greater than that of the water. Such a section should always be safe, therefore, although it may not be as safe as Levy thought it was on account of actual non-linear stress distribution toward lower elevations. When the base width is less than that prescribed by Levy, the dam is safe only provided the uplift does not exceed a certain definite amount.

The selection of uplift as 40% of the hydrostatic pressure, as advocated by the author, seems rather low; its intensity depends upon so many factors about which the engineer can know little. When the stresses due to earthquake shocks and the fact that nothing very precise is known about the stresses at the base are included, the conservative designer will hesitate to depart very far from the Levy section when designing an important structure.

The effect of uplift is well illustrated in Fig. 5. Fig. 5(*a*) probably represents a favorable case and Fig. 5(*b*) represents a case more severe than it would ordinarily be necessary to assume. The uplift pressure cannot act on the full base area, and the author chooses 40% of this area as the maximum. In order to inject a factor of safety into this uncertain matter, the late James B. Francis, Past-President and Hon. M. Am. Soc. C. E., recommended that the total base area be used for the calculation of uplift. Such a factor of safety seems very sensible.

The Levy assumption, represented by the case in Fig. 5(*b*) (except that $X = 0$) lies between the two extremes illustrated in Fig. 5(*a*) and Fig. 5(*b*); it therefore seems logical not to discard the Levy assumption when designing gravity dams of medium height, except for dams on the very best of foundation materials.

Since the effect of uplift is largely a matter of judgment, a lower and an upper limit of base width for gravity dams could well be decided upon by the profession for the guidance of State engineers or other regulatory bodies. The lower limit of base width would be for dams on excellent and tight

[28] "Experimental Investigations of the Stresses in Masonry Dams Subjected to Water Pressure," by Sir John W. Ottley and A. W. Brightmore and by J. S. Wilson and William Gore, *Minutes of Proceedings*, Inst. C. E., Vol. CLXXII, Session 1907-08, Pt. II, Lond. 1908; see, also, for example, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 505, and Figs. 8, 9, 12, 13, 14, 18, and 19.

[29] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 536.

[30] "Technische Mechanik," von Otto Mohr, Second Edition, p. 296.

[31] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 489.

foundations, and the upper limit would be for those on seamy rock not so solid. The judgment of the engineer would then decide each intermediate case. With concrete weighing 150 lb per cu ft, it seems reasonable to make these limits of base width 75% and 85% of the height of the dam, respectively, for dams of moderate heights of triangular cross-section.

It is true that a number of gravity dams have a base-to-height ratio of about 66⅔, and when zero uplift is assumed, no earthquake shocks are allowed for, and linear stress distribution is assumed, such sections have a factor of safety of one against overturning and it is quite immaterial what their factor of safety is against sliding, provided it is greater than one. In order to be able to show a factor of safety for such sections materially greater than one, it will be necessary to count on the shearing strength and the tensile strength of the concrete or bond to rock. To expand such a section to a base width of 0.75 *H* will require about 12½% more material and, perhaps 10% additional cost. This seems a small difference.

The Pine Canyon Dam on the San Gabriel River, in California, a dam of moderate height, has a base width of 86% of the height, and a substantial top width. Being built on good rock foundation (although not excellent), this dam is probably the most conservative gravity dam in the West, where there are too many gravity dams with base widths considerably less than 0.75 *H*, several of which, like the Pine Canyon Dam, are in earthquake zones.

In a dam actually developing arch action, conditions are most favorable to the utilization of the shearing strength of the concrete, as a wedge action is present and the shear surfaces are definitely under compression. The trough-like surface along the rock contact can then withstand considerable shear with safety, as mentioned by Mr. Henny, since shear action can be depended upon in combination with compression.

Most dam failures, in the past, could have been prevented by designing a heavier section. The behavior of the St. Francis Dam, as referred to by Mr. Henny, shows this clearly. The end portions of this dam, with base widths of 0.6666 *H*, promptly failed when the water level in the reservoir had nearly reached the top, whereas the middle portion of the dam, having a bottom width equal to about 82½% of the height stood. There was no shearing or tensile strength to help hold the cantilever down because there was no bond to the poor foundation. Such bond would have been necessary to keep the ends in place. There was no shear or bond strength available in the middle either, but this was not so necessary there on account of the heavier section. There is no substitution for weight in a gravity dam, except perhaps the addition of buttresses on the down-stream side, but so far this expedient has not been very popular. To the writer it would seem quite logical and inexpensive to increase the overturning resistance on dams of medium height in this way as no gravity dam has failed initially by the crushing of the down-stream toe.

The deflection curve for a triangular cross-section of a cantilever under water load is approximately a circle, and the maximum deflection at the top

of the triangle is $\frac{62.5\ H^5}{E\ L^3}$, using the author's notation. For such a section with $H = 200$ ft, say, the deflection at the crest would be four times larger than with $H = 100$ ft. For a dam in an inverted A-shaped canyon, the cantilever will deflect more in the middle at the crest than on the side-hill. The horizontal beam will attempt to neutralize this tendency by transferring the load from the middle high portion of the dam toward the shorter cantilevers on the side-hill. For equal safety the section on the side-hill, therefore, should be heavier than the section across the middle high portion of the opening. The writer mentions this specifically because little attention has been paid to this feature in the past, and it should always be the aim of the designer to produce a structure having the same factor of safety in all its parts. An arched gravity dam is worse in this respect, than a straight gravity dam, since the ends of the arched gravity dam generally have to be placed on a down-hill slope, and designers of gravity arched dams similar to the St. Francis Dam cannot rely on help from the arch at the season of greatest contraction (March).

The author does not advocate slimmer sections than those now in use, but it also seems that he does not point out the absence, to all intents and purposes, of a factor of safety in a gravity dam section with a base width of $66\frac{2}{3}\%$ of the height, in case the tensile and shearing strength of the concrete should fail through cracking or should be diminished with age.

A dam with such a section may be on the verge of failure, and 10% added to the cost of such a structure most likely will mean the difference of its success or failure. The St. Francis Dam was a practical demonstration of the behavior of a dam section with a base width of 0.6666 H in one place and a section with a base width of $82\frac{1}{2}\%$ of the height in another place. About 10% added to the cost of this dam could have expanded the base width of the ends (that failed) to $82\frac{1}{2}\%$ of the height at the rock contact, the same as for the middle portion, it would have deepened the excavation, and undoubtedly would have prevented the tremendous loss involved. The engineer must consider that in one case he is practically "flirting with the undertaker" and, in the other case, he is, under ordinary circumstances, producing a dam of adequate safety.

The Engineering Profession should establish an allowable minimum base width for a gravity dam and this should be considerably in excess of 0.6666 H. There would still be plenty of room for the exercise of judgment by the individual engineer in deciding how much more than the allowable minimum the base width should be for his structure.

I. M. NELIDOV,[32] ASSOC. M. AM. SOC. C. E. (by letter)[32a].—When a gravity dam is acted upon by the pressure of water, the resistance due to shear should be introduced in addition to the resistance due to sliding friction. This is the substance of Mr. Henny's paper.

[32] Senior Engr. of Hydr. Structure Design, State Dept. of Public Works, Sacramento, Calif.

[32a] Received by the Secretary December 5, 1933.

The formula given for total resistance per unit of area is: $s = s_1 + kq$, and a factor of safety for the entire horizontal section of $Q = 4$ is adopted. Uplift loading, including the variation of its intensity with vertical loading, and the area of its application are also discussed.

It must be recognized that for about fifty years there has been little advance in the knowledge of the proper design for the stability of structures in general and of dams in particular. The importance of obtaining reasonable values for stability factors should not be minimized, especially if comparison is made with those of stresses. For instance, with compressive stress in concrete of 500 lb per sq in., the factor of safety is at least 4. At the same time, while computing stability against horizontal forces, with the usual sliding ratio of about 0.6 and disregarding shear resistance, a safety factor of only 1.3 is obtained. A safety factor in overturning equal to 1.5 is considered very satisfactory in case the resistance due to bond with the foundation is not taken into account. With uplift both factors attain still lower values. This indicates to what extent stability factors are smaller than those of stresses. On the other hand, raising stability factors to the same value as stress factors would result in excessive dimensions, provided no resistance other than sliding friction and the resisting effect of dead weight is included.

As shown in the paper the inclusion of shear resistance gives an increase of safety factor from about 1.3 to 4, thus indicating that the greatest part of the resistance is supplied by this newly included element. This condition requires a serious study of these new elements, since the safety of the dam depends entirely on the correctness of the values selected. An investigation of the ultimate resistance due to shear, sliding, and bond should be made for planes of contact of concrete with rock, at construction joints, and within the material itself. The fact that the rock structure is not uniform and that planes of weakness may exist hidden from the investigating eye, must be taken into account. This, together with saturation of the foundation rock, should warn against the haphazard use of formulas including resistance due to shear and bond. Sound design should be based upon carefully selected test data and reasonable safety factors approaching 4 should be attained. It is wasteful to neglect the resistance if it can be relied upon. During the construction of Hogan Dam in 1929, the writer used data on shear resistance of horizontal joints obtained by Mr. L. H. Tuthill,[33] but was very cautious in applying them due to the singularity of the test. The fact that stability can be altered so greatly by the inclusion or the omission of the shear element seems to justify this attitude.

The process by which the values, s_1 and k, are obtained is necessarily approximate. First, the assumption of a plane of rupture is far from what actually occurs in a cylindrical body; and, second, the coefficient of friction is not constant with normal stress. This makes Equation (4) an unknown of a high degree. Professor S. Timoshenko[34] states that if the surfaces of

[33] "Horizontal Joint Shear in Mass Concrete," by Lewis H. Tuthill, *Western Construction News*, November 25, 1931.

[34] "Strength of Materials," by S. Timoshenko, Pt. II, p. 655.

contact of concrete specimens are covered with paraffin, the ultimate strength will be greatly reduced and the type of failure will be completely different. A solid cube fails by dividing into planes parallel with one of the lateral sides. This indicates that the mode of loading will change the character of failure entirely. Besides, the actual condition of loading is far from that produced in a laterally unsupported cylinder as used in concrete tests. Consequently, the values of resistances, s_1 and k, are not representative and may not even be correct. It appears that tests similar to those by Mr. Tuthill, in which a middle block was displaced relative to the side blocks, should supply more correct values for shear and sliding resistance. The segregation of shear and sliding proper can be made by testing the displaced specimen for frictional resistance only.

Experiments quoted by the author and those by Mr. Tuthill indicate that total horizontal resistance increases with the normal pressure to a certain value, beyond which it begins to fall. Crushing of particles may be responsible for this fact. This may also raise a question concerning the simultaneous action of shear and sliding. A negative answer to this question would disprove the theory based on the combined action. Additional experimental data will supply the answer.

In passing, the writer wishes to remark that it would appear logical to use formulas for stresses which take into account tension in concrete when the total width of the base is taken in computing sliding-shear resistance.

To complete a study of stability, overturning also must be discussed. Assuming no bond between the base of the dam and foundation, the stability is assured only by the resisting moment of the weight and other vertical forces acting downward. In a general case there may be four sums of forces affecting the stability in overturning, namely, the vertical forces acting downward and upward and two horizontal forces acting to the right and to the left. The first sum, W, results from the weight of masonry, vertical water pressure, and weight of débris on the up-stream and down-stream faces; the second, U, is produced by uplift; the third, P, by the horizontal water pressure and débris on the up-stream face; and the fourth, P_1, by the horizontal water pressure on the down-stream face. The factor of safety in overturning, as in the case of the factors of safety in sliding, tension, shear, etc., must express the ratio of the ultimate overturning ratio (at which overturning is bound to occur) to the overturning ratio produced by the acting forces in the case considered. Overturning ratio is defined here as:

$$r_o = \frac{\Sigma M_r}{\Sigma M_o} > 1 \qquad (20)$$

in which, ΣM_r is the moment that resists overturning; and, ΣM_o is the moment that produces overturning. At the point of overturning the ultimate overturning ratio is:

$$r_u = \frac{M_r}{M_u} = 1 \qquad (21)$$

in which, M_u is the moment at which actual overturning occurs.

The factor of safety against overturning will be:

$$F = \frac{r_o}{r_u} > 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

According to the definition of N. Kelen[35]: $M_r = M_w + M_{p1}$; and, $M_o = M_p + M_u$; or,

$$F = -\frac{M_w + M_{p1}}{M_p + M_u} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which, positive moments are clockwise.

If a dam is anchored to its foundation, or if bond between the concrete and rock can be relied upon, these new elements will increase the potential resisting moment within the limits of their own factors of safety. Consequently, consideration should be given to the existing and allowable factors of safety of the anchorage. When the actual factor of safety of the anchorage is smaller than the allowable, the anchorage will be unreliable until the stress in it will be lowered to such a value that the factor of safety will become equal to the allowable factor. For example, using the author's notation, select a dam with $H = 150$ ft; $L = 113.4$ ft; $n = 0.755$; and a $= 2.4$, which corresponds to the curve in Fig. 8 with $Q = 4$.

For the condition of no uplift: $l_1 = 42$ lb per sq in.; $l_2 = 114$ lb per sq in.; and $F_1 = \frac{M_w}{M_p} = 2.74$. If the ultimate bond stress is taken at 150 lb per sq in., then, with a factor of safety of 4, only 37.5 in-lb may be allowed for safe tension. The additional overturning moment which will bring the stress to this value will be:

$$M'_o = \frac{1}{6}(37.5 + 42)\,144 \times 113.4^2 = 24\,659\,000 \text{ ft-lb}$$

The new factor of safety will then be:

$$F_2 = \frac{M_w + M'_o}{M_p} = 3.44$$

For the condition of 50% uplift at the up-stream face (32.5 in-lb) and 0 at the down-stream face: $l_1 = 42 - 32.5 = 9.5$ in-lb, and $l_2 = 114$ in-lb, which assumes that bond between the concrete and the rock was not broken by the uplift force. Disregarding the reserve due to the bond, the factor of overturning will be:

$$F_1 = \frac{M_w}{M_p + M_u} = 1.74$$

The additional overturning moment that will bring the stress to the maximum allowed (37.5 in-lb) will be:

$$M'_o = \frac{1}{6}(37.5 + 9.5) \times 114 \times 113.4^2 = 14\,505\,000 \text{ ft-lb}$$

[35] "Die Staumauern," von N. Kelen.

This will make the factor of safety, including bond, equal to:

$$F_2 = \frac{M_w + M'_o}{M_p + M_u} = 2.00$$

If Mr. Henny's assumption is followed in that the bond between the concrete and the rock is broken by uplift, the stresses will be determined by Equations (10) and (11) thus: $x = 100.2$ ft; $l_2 = 300$ in-lb; and $l_1 = 0$. In this case:

$$F_1 = \frac{M_w}{M_p + M_u} = 1.02$$

The foregoing computations indicate that factor in overturning may be one of the weak spots in stability. The writer wishes to express his appreciation to the author for the study of this apparently simple but little discussed problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ESTIMATING THE ECONOMIC VALUE OF PROPOSED HIGHWAY EXPENDITURES

Discussion

BY J. T. L. MCNEW, M. AM. SOC. C. E.

J. T. L. MCNEW,[22] M. AM. SOC. C. E. (by letter)[22a].—The author has long been giving advice to the profession as to the economic aspects of highway improvement, but only in recent years have any road users allowed the engineer to improve roads only to the extent directed by such economics. As long as road users' demands were insistent that more miles of pavement be laid, and they did not complain of the tax burden, few were the engineers who would direct attention to such subjects as "justifiable expenditure," "savings in transportation costs," etc.

With the débacle of 1929 and its effect upon tax payments, the diversion of road toll levies for poor relief, the talk of road-building holidays, and the passing of all bond markets, the stage has been re-arranged and the scenery changed to cast the remainder of the road construction projects on a more sound financial procedure.

Perhaps the slowness with which highway economics has proceeded has been due, in a measure, to the slowness with which a National road-building program was undertaken. The accentuated demand for public roads began about 1900, when the automobile made its début, but it was about sixteen years later before the United States embarked upon a program of Federal roads. By 1916 the demand for roads was so insistent that it could not wait and for the next thirteen years nothing mattered much as long as the roads were paved, and the sooner the better. A perusal of typical publications of about 1908 will show that first the public had to be "sold" on the idea that good roads were a benefit and, thereafter, the people bought wholeheartedly and unstintingly.

With about two years now (1932 and 1933) in which to pause and reflect on the need for roads, one must quickly see that no longer can the public

NOTE.—The paper by Thomas R. Agg, M. Am. Soc., C. E. was presented at the meeting of the Highway Division, Atlantic City, N. J., October 10, 1932, and published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. W. W. Crosby, Roger L. Morrison, and Samuel B. Folk; and December, 1933, by Messrs. W. S. Downs and George E. Martin.

[22] Prof. of Highway Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[22a] Received by the Secretary November 13, 1933.

bear expenditures on capital improvements unless the tax levied and collected will be returned to the taxpayer either in whole or in increased amounts. Perhaps the financial return may be slightly less at times than the tax burden, and the increment of difference will represent an elevation of the standard of living; however, communities that recognize such a principle are few in this day when standards of living are apparently below the peak of former years.

Perhaps the alacrity with which the public has provided road construction funds may be due to the typical American custom of buying a thing, and then dismissing the matter. In other words, the public has been permitted to think that once a road was paved it would be paved permanently and, with such a thought, the cost price was provided. Now that the probable life cycle of every type of pavement has been about reached or passed since the beginning of road policies in the United States, it is about time all engineers "took a hand," and showed the economic facts that justify road improvement, and refuse to countenance programs not based upon such facts.

Practically all expressions for annual cost involve the term, residual value or salvage value, for instance, indicating that one stage of construction may well be a valuable part of the next type of improvement, and yet this one factor is often overlooked in so-called Unit-1 construction to such an extent that when a road type is changed from a gravel surface to a higher type, it becomes necessary to change the grade so much that the salvage value of the gravel becomes zero. In such cases, unless traffic type has changed radically, there is an indication of forgotten economy. In recent years the researches of the U. S. Bureau of Public Roads have shown that certain sub-grade soils, namely, Classes A-6 and A-7[23] are probably unsuited as sub-grades under high types of pavements and that the sand-clay and gravel mixtures are capable of providing valuable sub-grades. It is probably true, then, that by planning road improvements more than one step ahead the engineer may actually reduce the loss on each stage of construction and, at the same time, have each stage justified.

The author's analysis of the change in cost of transportation on United States Highway 65, resulting from a change from a gravel to a high type of surface, offers an example of the point the writer desires to make. Under the cost analysis of the gravel surface, the author shows the entire value of the surface to be charged against the gravel surface and that as a sub-grade for the higher type the old gravel has zero value. Is not this treatment indicative of either an ill-chosen grade line for Unit 1, or a rather severe charge to the gravel type? If it is assumed that all the gravel is bladed to the shoulders, is not then the traffic-compacted sub-grade of some value in excess of that of the freshly constructed sub-grade, and would not the value of the gravel in the shoulders be the equivalent of a measurable salvage value? In many cases the writer feels that the salvage value of an old surface, when used as a foundation for a high type of pavement, may be even more than the original surface cost, under certain conditions.

[23] *Public Roads*, Vol. 12, No. 5, July, 1931, p. 136.

The economics of highway location with respect to distance saved or to the cost of operation over excess distance is another phase of the general problem that has caused much debate. Capitalizing theoretical savings of transportation due to shortening of distance offers many pitfalls, and yet it is a logical part of any cost analysis. The writer doubts the wisdom of setting up figures to justify savings of distance unless those figures are chosen with conservatism. Is an engineer justified in assuming that any average road, located with reasonable care and skill, will for all time to come be the right location, or that it will offer the same service as it does when it is built? Unless the answer is in the affirmative, one may say that the justification for relocation may change and when that change is demanded the savings will no longer pay the interest on the first cost of the improvement, especially in cases where the amount first computed as justifiable actually has been spent. The writer recognizes that the cost of extra distance may be tremendously large in some cases, but he doubts very much if the user of the road who actually accumulates the savings appreciates the fact, and he also doubts that the ones who do accumulate the savings would ever be willing to donate even one-fourth of such savings back into construction funds as a road tax.

Dean Agg has computed the justifiable expenditure for saving distance by first assuming that the saving will be effective for twenty years. This is a noteworthy change from the methods used to justify saving distance as often published. Twenty years may be a long time in numerous cases, and the writer agrees with the author that "comparisons of this type should be used with great caution.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE SURVEYOR AND HIS LEGAL EQUIPMENT

Discussion

BY MESSRS. CLARENCE T. JOHNSTON, A. F. HARLEY, AND LYNN PERRY

CLARENCE T. JOHNSTON,[94] M. AM. SOC. C. E. (by letter)[94a].—The author has brought together in compact form many details of the law relating to boundaries which concern the surveyor and influence his professional activities. Volumes pertaining to the same subject have been compiled by authorities in the law. Several works have been written by men who represent both legal and scientific opinion. Those who specialize in boundary surveys have long felt the need of a digest of the essential element of the law. To select the important features from the accumulated mass, represents a task which, to resort to paraphrase, makes surveyors lose the good they might otherwise win by fearing to attempt.

The practical experience and special interests of surveyors influence their judgments regarding the relative importance of specific provisions of the law. Many surveyors chafe under the limitations that the law seems to prescribe. If the surveyors who made the basic surveys had been adequately supported at the time such surveys were made, much of the existing law would now have but little weight or meaning. Specifications defining enduring monuments are not to be found in the common law. The surveyor is often ready to condemn much of this law as it relates to his work because it ignores science and because it does not refer to standard monuments, limiting closing errors, and other items of major importance to him.

As the essential features of the law are brought together and discussed by those who have conflicting opinions, a foundation should be laid for a common understanding among surveyors. Professor Holt's paper is sufficiently comprehensive to form a basis for a national discussion of the law relating to boundary surveys. When the discussion is completed and final conclusions are drawn, the essential features of the law should be outlined

NOTE.—The paper by A. H. Holt, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. Ray H. Skelton, William Bowie, R. Robinson Rowe, and Walter H. Dunlap; and December, 1933, by Chester Mueller, Assoc. M. Am. Soc. C. E.

[94] Prof. of Geodesy and Surveying, Univ. of Michigan, Ann Arbor, Mich.

[94a] Received by the Secretary November 24, 1933.

and recorded in terms that are easily understood. This may lead to the enactment of statutes which reflect the best of modern practice.

The law relating to boundary surveys has not improved to keep pace with the growth of scientific theory, reformed practice, and better equipment. Science has long since prepared the way for the introduction of a new era in boundary surveying. Surveying and land tenure began to be recognized when early peoples departed from nomadic customs and engaged in occupations that involved settlement and the cultivation of the soil. Science was in its infancy when lands were parceled out to individuals and families. The rules governing the occupation and use of lands became tribal traditions. They were finally incorporated in the "unwritten law" and modified to meet the necessities of many early peoples. The law continued to grow as life became more complex. Science had but little application to boundary surveys until long after the development of comprehensive codes relating to land tenure. The power to make final decisions was then, as now, entrusted to those trained in the law. It is not a matter of accident that there are volumes of legal discussion relating to the boundaries of real property and a scanty literature dealing with the applications of science to the same subject. The process of evolution is extremely slow as traditional doctrine and opinion undergo modification to meet, in a small way, the requirements of a changing society. As specifications based on science replace legal authority, the law relating to boundaries must be simplified.

Surveyors are obliged to familiarize themselves with the law as it exists because the power to establish boundaries rests with the Legal Profession and the Court. There are unmistakable signs of agreement among surveyors as to the logical first steps they should take to hasten reform. A review of boundary surveying in the United States discloses an almost uniform carelessness regarding monuments. Distinctive, enduring monuments marking the intersection of boundaries should be the objectives of every boundary survey. While the quality of work performed in connection with many original surveys was not high, errors in linear and angular measurements and discrepancies in the official records may well be overlooked. Carelessness relative to monuments cannot be condoned. The cost of distinctive, enduring monuments is trifling compared with the total cost of public or private surveys. Failure to set such monuments evidences a lack of appreciation of the first objective of a boundary survey.

It is recommended, therefore, that standard monuments, such as are used by the General Land Office, the United States Coast and Geodetic Survey, and other departments of the Federal Government, be set to mark the intersections of real property boundaries in all future surveys. Distinctive, enduring monuments will be seen and respected. Freeholders, and the public generally, will become boundary conscious. An appreciation of good monuments should soon lead to triangulation control, local systems of co-ordinates, and complete records of all surveys in appropriate public offices.

The surveyor who is retained by one or more freeholders to set monuments and thus mark real property boundaries, has responsibilities which are often

overlooked. In the first place, he should have the technical qualifications that are recognized by the leaders of his profession. He should be familiar with the law as it is outlined by Professor Holt and by those who make positive contributions to the discussion of his paper. He should understand that his work should be performed in such manner as to merit the confidence of his clients and the respect of other interested parties. Whether the locations of monuments are determined by agreement or by order of the Court, he should urge the setting of standard monuments. He should understand that all boundary surveys are matters of local public concern, and he should leave a plain and complete record of his work in some designated public office. A surveyor who regards the notes and computations to boundary surveys as private property does not exhibit a professional attitude toward his work. In the long run his own practice suffers as his competitors pursue the same selfish and short-sighted policy.

In his concluding sentences Professor Holt refers to the value of "definite, distinctive, and enduring monuments." If all real property boundaries were to be marked by such monuments, the public would be afforded the basic protection that surveys should afford. The setting of such monuments would introduce a new era in boundary surveying.

A. F. HARLEY,[95] M. AM. SOC. C. E. (by letter)[95a].—The analytical nature of this paper is its principal charm, and its publication becomes at once a graceful introduction to the works of Clark and Skelton.

At the very outset of the author's remarks one quickly discerns the difference between a lost and an obscured corner. In ninety-nine cases out of one hundred, "lost" corners are merely obliterated corners, and their finding is as interesting as a jig-saw puzzle.

Before 1912, however, the writer did not think so, and was prepared to swear that each obliterated corner in cut-over pine woods was in every sense of the word a lost corner, since all the old trees, "witnessed," "blazed and hacked," or merely good round timber had been cut and carried to the nearest saw-mill. Then he happened to read a paper[96] by N. B. Sweitzer, Assoc. M. Am. Soc. C. E., describing a method of finding the original pits surrounding a section corner mound, which had been filled in, and leveled down by the wind, in the flat, dry, desert soil of Nebraska, and concluded, that in comparison the task of finding obscured corners in the cut-over pine woods of Florida was comparatively an easy one. This it not only proved to be, but it held a Judge of the United States Court in close attention, until the method pursued had demonstrated its unerring precision.

The writer's method is as follows: Approximately, at an obscured section corner a stake is driven, the transit set up, the vernier set on zero, the telescope pointed toward the north, and azimuth angles and stadia distances are read and recorded to every stump within a radius of 200 ft, or more, from the instrument.

---

[95] Cons. Engr., Jacksonville, Fla.

[95a] Received by the Secretary November 25, 1933.

[96] *Transactions*, Am. Soc. C. E., Vol. LXXV (December, 1912), p. 393.

The rodman is previously instructed to hold his rod on every stump, or what may remain of the tap-root of a tree, or at the center of surface lateral roots, running out from where an old stump had been, and to cut a chip from each, before going to the next stump. This done, he is again told to walk backward and forward over the ground and especially through high grass and briars to look for other stumps he may have overlooked. These angles and distances are then platted in the office, and each stump is shown by a black dot; the drawing is finished with a north point.

The witness trees of the particular section corner, as marked by a United States Deputy Surveyor and recorded in his field notes under instruction from Robert Butler, Surveyor-General of the then Territory of Florida, in 1848 and 1849, are then platted on a piece of tracing paper, and the four or less witness trees shown by small circles; and this drawing is also finished with a north point.

Then commences the jig-saw puzzle, until a spontaneous exclamation from the draftsman of "I have it" indicates that while keeping the north point on the tracing paper parallel with the north point on the drawing beneath it, and moving the tracing paper up and down and from side to side, four black dots suddenly appeared within the four circles.

Fig. 1 is an example from the public records of Duval County, Florida, showing this indisputable evidence of the true position at which 8 by 8-in.

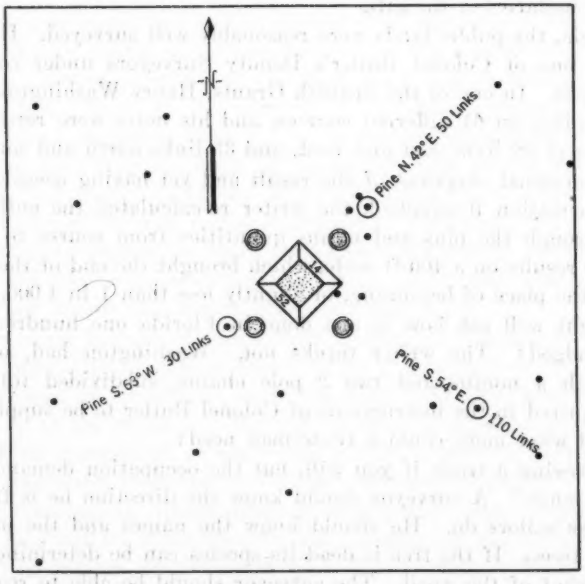

FIG. 1.

concrete monuments have been placed to mark for all time certain heretofore obscured section corners and quarter-section posts in Duval County. The dots represent the trees or stumps of trees; the dots enclosed in a circle are

the trees or stumps of trees the bearing and distance of which conform to the bearing and distance of the witness trees as recorded in the field notes of the U. S. Deputy Surveyor when surveying public lands, and now preserved in the State Capitol at Tallahassee, Fla.

Fig. 1 also shows the 8 by 8-in. concrete monuments (drawn to a larger scale), turned so that the section numbers cast on the side of the concrete monuments may face the sections to which the numbers refer. It also shows four shaded circles which represent four small circular concrete monuments, buried entirely below the surface. These witness monuments would enable a section corner monument that had been moved, to be replaced in its original position, or they would enable a surveyor to substitute a new section corner monument for one that had been broken.

Of a truth, the men engaged in this work had found the footsteps of the surveyor; and more than this, they actually knew, after the final transit lines had been run from monument to monument, the average error in measurements the original surveyor had made under his contract with the Government on a per mile basis of compensation.

Cost? Oh yes, it costs, but Uncle Sam did not learn from his predecessor, who was equally generous in giving homestead lands away to the early settler. In California, the receiver of a gift of land from Spain was heavily fined if within a reasonable time thereafter, properly laid-up stone monuments did not mark the boundaries of the gift.

In Florida, the public lands were reasonably well surveyed. Henry Washington was one of Colonel Butler's Deputy Surveyors under contract, and one of his best. In one of the Spanish Grants, Henry Washington ran 75 450 ft, or 14.3 miles, on 61 different courses, and his notes were returned with a closing error of 32 links east and west, and 35 links north and south.

Being somewhat skeptical of the result and yet having occasion to use it for the information it supplied, the writer re-calculated the entire traverse, carrying through the plus and minus quantities from course to course and platting the results on a 400-ft scale, which brought the end of the last course 72 ft from the place of beginning, or slightly less than 1 in 1 000.

One might well ask how it was done in Florida one hundred years ago (1834). Fudged? The writer thinks not. Washington had, of course, a compass with a nonius and two 2 pole chains, subdivided into 50 equal links, as required in the instructions of Colonel Butler to be supplied by each Deputy, and what more could a tradesman need?

Call surveying a trade if you will, but the occupation demands considerable "horse sense." A surveyor should know the direction he is facing at all times just as sailors do. He should know the names and the probable life of different trees. If the tree is dead its species can be determined from the grain and scent of the wood. The surveyor should be able to recognize new growth over an old scar, and should know how to remove such new growth and expose the old scribing. He should know by rote the instructions of the Surveyor General appointed by Congress to conduct the survey of an acquired territory, which were issued to the Deputy Surveyors in the field, thirty years

before the first Manual published by the General Land Office ever left Washington, D. C., and he should also know the *modus operandi* of that Surveyor General in carrying out the orders of Congress. Knowing these things he will have learned the first three letters of the alphabet of the surveyor's trade.

LYNN PERRY,[97] M. AM. SOC. C. E. (by letter)[97a].—The subject-matter of this paper is timely, and the author is to be complimented on the thorough and interesting manner in which he has handled a subject so difficult to present clearly. The paper is well authenticated and well annotated. Although many of the citations appear to be somewhat local, the legal principles laid down by the Courts in the excerpts cited are general, and the inexperienced surveyor will do well to follow them.

Many engineers, at some period during their professional career, become involved in boundary disputes and property lines. To some, this duty comes at a time when they are least adequately prepared, legally, to render the decision that is expected of them. The attention of the writer was drawn to the logical legal evaluation of the "calls" in the description of a property, early in his career. He was surveying a tract of land in Maryland, the northern boundary of which was described as "thence, due east" a certain distance "along the Mason and Dixon line dividing Maryland from Delaware." The Mason and Dixon line at this point was monumented in 1760, after about nine years of surveying, injunctions, agreements, Court decisions, etc. More recent researches have disclosed that Mason and Dixon probably did not inspect the field work of that part of the line themselves; they merely approved the manner in which it had been done and authorized the line to be monumented. Much to the surprise of the immature surveyor, it was found that the monuments were not in an easterly line (in fact, they were not in line at all); they were not at 1-mile intervals, except approximately; and the east and west ends of the client's holdings in no way corresponded with the distance mentioned in the description. Obviously, the property held under a Maryland patent for more than 140 years was in no way a part of Delaware, and the northern boundary was made, not due east, but a meandered line connecting the existing monuments.

An interesting feature of this survey is that one monument was missing and was found being used as a door step by a near-by tenant farmer. The man had no idea of its history, as it had been used as a door step prior to his occupancy of the property. He was finally persuaded to acquiesce in the resetting of the monument at the location from whence it had been moved. This point was fairly definitely settled by surface conditions.

Another interesting illustration of this pitfall into which a capable technician may fall is illustrated in the resurvey of the circular boundary separating the States of Delaware and Pennsylvania. This is the only mathematical curve forming a boundary that has been brought to the writer's attention. In 1849, nearly 90 years after the northeast corner of

[97] Asst. Prof., Hydr. and San. Eng., Lafayette Coll., Easton, Pa.

[97a] Received by the Secretary December 2, 1933.

Maryland was originally monumented by Mason and Dixon, and 148 years after the original circular boundary between Pennsylvania and Delaware had been run by Tailer and Pierson, the three Commonwealths interested, by proper legislation, set up a commission to re-run and re-mark the northeastern corner of Maryland and the circular arc forming the northern boundary of Delaware.

This survey relocated the northeastern boundary of Maryland and during the excavation incident to the erection of a permanent monument, the old monument set by Mason and Dixon was found about 4 or 5 ft below the surface. The resurvey checked within about 2 in. This was well, but when the surveyor began the resurvey of the circular arc, he found some small mathematical errors in the Mason and Dixon computations and linear measurements. He surveyed the way he thought Mason and Dixon should have surveyed, following the original grants from Charles I, Charles II, and James II, and the subsequent agreements down to and including that of Frederick, the sixth (and last) Baron of Baltimore, and the heirs of William Penn. He must have done a tremendous amount of research work prior to going into the field because his report is found to be correct in every detail, and his geodetic work carefully and accurately performed. He overlooked the fact, however, that he was engaged to make a resurvey and not to establish a boundary. The result was that he took from Delaware a small triangular shaped parcel of territory, several miles long and less than 2 miles wide, and gave it to Pennsylvania. The result was that Pennsylvania, standing to benefit by the increase of territory and population, promptly approved the survey; the Maryland General Assembly not being interested in the dispute, and being contented with that portion of the survey which concerned that State's boundary, ratified the survey; but Delaware never ratified it. All the maps of this locality prior to about 1900 will show Pennsylvania (Chester County) holding this small wedge of territory extending down between Delaware and Maryland. Pennsylvania, however, was never able to collect the taxes nor in any way to exert jurisdiction on this triangle. For many years after dueling was made illegal by law, duelists sought this location for final settlement of personal disputes. Only within the past decade has jurisdiction over this tract of land been settled finally.

Maryland has a boundary dispute with West Virginia. The original grant of Maryland explicitly states that its western boundary shall be a meridian line running south from the Fortieth Parallel to the most distant (farthest from Chesapeake Bay) fountain of the Potomac River. This "most distant fountain" is known and has been known many years to be the South Branch of the Potomac River and the boundary, if properly run as originally intended, would give Maryland several counties now in West Virginia. The validity of Maryland's equity to this territory is admitted in the Constitution of Virginia adopted June 29, 1776 (Article 21), the first Constitution Virginia ever had. Due to the many Virginia patents, to the objection of many inhabitants, and somewhat to procastination, Maryland was not able to consolidate and bring this territory under its jurisdiction. In 1863, it became

West Virginia. What surveyor would now make the western boundary of Maryland conform to the original intentions of the grantors and grantees?

The surveyor must bear in mind that he is not a judge. He is an expert witness to whose testimony the Court will give due weight if his work has been characterized by care and judgment. When his decisions do not conform to the law, as laid down in the decisions of higher Courts, his technical care may be of little value to his client.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PHOTO-ELASTIC ANALYSIS OF STRESSES IN COMPOSITE MATERIALS

Discussion

BY MESSRS. H. J. GILKEY, AND ROBERT V. BAUD

H. J. GILKEY,[7] M. AM. SOC. C. E. (by letter).[7a]—In extending the application of stress analysis by polarized light to composite materials new techniques have been developed and a new range of possibilities has been tapped. For these reasons, if for no others, the authors are deserving of commendation. A reconnaissance is usually suggestive and among the interesting further extensions that appear possible are:

1.—Studies in the transfer of stress from one bar to another as in the vicinity of a column splice. Such studies could include different types of splices and should certainly include, as a variable, the distance between the bars and the relative volume of the matrix through which the transfer of load must be accomplished.

2.—More complete experimentation with the hooked bar with reference to such factors as the radius of the hook and its relationship to the sequence of failure; that is, whether the matrix (celluloid or concrete) crushes under the bend; whether the hook "snakes" out of its curved hole; or whether it splits the block in which it is embedded.

3.—Comparison of a few large bars with many small ones to transmit a given total stress.

4.—Studies of type shapes of deformed bars, that is, those with wedge-like lugs as against those with square shouldered lugs. Such studies could also include studies of polished, etched, coated, tapered, and anchored bars.

5.—Closely allied to Item 4 would be studies of bond stresses in the vicinity of cracks and the extent to which the welded cross-wires of a mesh function as lugs.

NOTE.—The paper by A. H. Beyer, M. Am. Soc. C. E., and A. G. Solakian, Assoc. M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[7] Prof. and Head, Dept. of Theoretical and Applied Mechanics, Iowa State Coll., Ames, Iowa.

[7a] Received by the Secretary, October 23, 1933.

6.—It might even be possible to study the action of hoops or spirals in resisting bursting tendencies.

7.—Perhaps future studies will develop other optical materials of properties such that the nature of the stress response can be made visible in each of the materials

8.—Perhaps beams can be weakened on the tensile side by means of saw-cuts, or otherwise, to simulate more perfectly the tensile part of reinforced concrete members.

Whether polarized light is the logical medium to use in a search for further truth on any such problems is quite another question and should be considered carefully before an elaborate program is launched in a particular case. It must be kept in mind that:

(*a*) At best, optical methods are essentially qualitative.

(*b*) The relationship between the two materials studied differs in a number of important respects from those for the types of materials about which information is desired.

(*c*) Thus far at least the evaluation of parts of the test results is by elimination, and is necessarily indirect.

(*d*) In many cases the qualitative nature of the action is already sufficiently well known, and the important fields to be covered are to obtain numerical evaluations by experimentation with members, or models of them, made up of actual materials or of materials that are similar to the actual materials.

(*e*) In other cases more thorough-going mathematical analyses will promise a more positive result and will net a greater return for the effort involved.

In spite of the questions which have been raised, the structural analyst will do well to keep these new possibilities in his consciousness, in order that he may have one more tool available if a specific need arises.

The clarity of the photographs seems to minimize one difficulty which some have recognized in considering the possible usefulness of polarized light for such studies as these. It is obvious that a stressed bar anchored by bond to a matrix, which completely surrounds it, is essentially a three-dimensional problem and, as such, would not be amenable to optical study. In other words, the bar is surrounded with successive layers or cylinders of stress. However, so much more of the cylinder walls is encountered as the edges are approached, that the direct penetration that occurs in the central part of the field offers a relatively slight obstruction to the light rays and does not appear to muddle the view as badly as it might reasonably be expected to do. Undoubtedly, however, there must be some drawing in or narrowing of the zone because of this three-dimensional aspect.

ROBERT V. BAUD,[8] ESQ. (by letter).[8a]—In the first part of this paper the initial stresses in reinforced beams and plates are discussed. Unfortunately, the authors have not provided the bands shown in the illustrations of this

[8] Beaver Falls, Pa.

[8a] Received by the Secretary October 28, 1933.

part with fringe numbers, so that the views are of no practical value to the reader, because successive bands do not necessarily denote a corresponding increase in stress, such as Curve *A*, in Fig. 13, but may mean any kind of stress distribution, such as Curves *B*, *C*, etc. The stresses in Fig. 7 are

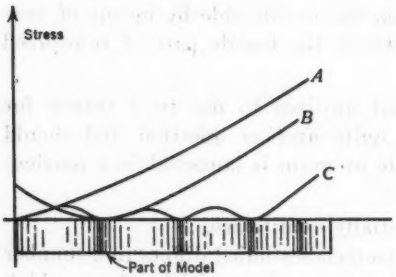

FIG. 13.

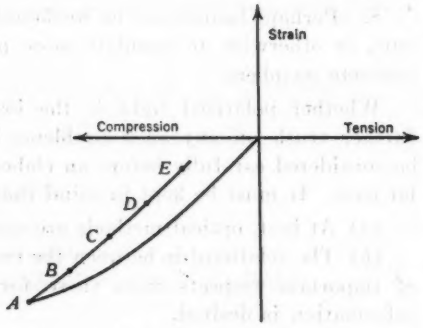

FIG. 14.

doubtful if Poisson's ratio differs for the two materials. In such a case a three-dimensional stress distribution is produced around the reinforcement rod and the stress readings are certain to be inaccurate.

It is desirable that the authors explain the percentage error in the last column of Table 2. One possible explanation is the probability that the principal stress directions for the initial condition are not all exactly parallel to the axes of the beam and that, therefore, they partly differ from the stress direction lines due to loading. If such is the case the initial field cannot be deducted algebraically, but must be subtracted according to the laws of tensor-fields. The most probable cause for the discrepancy in actual and computed moments, however, is the fact that the aluminum rod was considerably overstressed initially. The requirement that $\int_F f\,df = 0$, yields an initial compressive force in the rod of $A = 100$ lb, and the requirement that $\int_F f\,y\,df = 0$, yields a force of $A = 92.8$, or an average of about 96 lb, corresponding to an initial compressive stress of about 50 000 lb per sq in. Due to loading the beam this large initial compressive stress was reduced, but for no bending moment used in the experiment was there actually a tensile force in the rod, as is schematically shown in Fig. 14.

More important and more disturbing than the lack of agreement in the actual and computed moments (at least, for the practical man) is the fact that large initial tensile stresses are set up. Particularly for small loads the gain of shifting the neutral axis downward is more than offset by the presence of these large initial tensile stresses, and the question arises as to what this condition will be for actual materials. The authors believe that the configuration of the initial stress curve (Curve *A* in Fig. 9) is the same and

that only the stress scale is different. If this is the case the only factor necessary to investigate further is the force, A, in the steel rod. To obtain this, one can proceed as follows: Cut a cement beam at numerous places at right angles to the rod and measure the initial and final length of the rod before and after cutting; from the difference in length the force, A, can be computed in the known manner.

As far as the optical part of the paper is concerned the writer is much pleased to note that the authors have so completely adopted, in this and other investigations, his method of simplified stress measurements.[9]

The second last sentence under "Introduction" conveys the impression that nothing of consequence has been published in this country on the subject of photo-elasticity except the papers referred to. This is misleading, because a considerable number of contributions have been made here since 1920 on the subject of the phenomena involved, as well as on the instruments used, which, as far as the measurement of the maximum shear stress is concerned, do not differ much, fundamentally, from each other.

From Table 1, any one not so well acquainted with the method could conclude that the colors of the orders are the same; this is not the case as a study made several years ago revealed.[10] The authors further state with regard to Fig. 1 that it "is a black and white photograph of the stress fringes taken with circularly polarized white light." This is an erroneous statement; the truth is that only one ray of the white light spectrum is circularly polarized, the others being elliptically polarized.[11] Unless the principal stress directions in model and comparison strip coincide a small error in the stress reading is introduced.

Credit is due the authors for adding a new group of problems to the already large number in which photo-elasticity proves its usefulness.

---

[9] "Further Development in Photoelasticity," *Journal*, Optical Soc. of America and Review of Scientific Instruments, Vol. 18, No. 5, May, 1929, p. 422.

[10] "The Analysis of Colors Observed in Photoelastic Experiments," by R. V. Baud and W. D. Wright, *Journal*, Optical Soc. of America, Vol. 20, No. 7, July, 1930, p. 381.

[11] "Contribution to Study of Effect of Elliptical Polarization upon Energy Transmission," by R. V. Baud, *Journal*, Optical Soc. of America, Vol. 21, No. 2, February, 1931, p. 119.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DURATION CURVES

Discussion

BY MESSRS. C. R. PETTIS, AND W. G. HOYT

C. R. PETTIS,[12] M. AM. SOC. C. E. (by letter)[12a].—A beautiful and valuable method is introduced in this paper, which can be used in the solution of certain engineering problems. Every engineering method that is assumed to be of value, possesses certain advantages. It likewise has certain limitations, and its full value cannot be realized unless the limitations are thoroughly understood.

In presenting a new method to the Engineering Profession it is especially desirable that the limitations be clearly stated; otherwise, certain engineers, who are not entirely familiar with the subject-matter, will be led into error in its application. For example, it is important to emphasize the author's explanation (see "Plotting in Terms of Mean Flow") that the run-off record at a point, *A*, on the stream may be used to represent conditions at some other point, *B*, on that stream for which there is no available record, by plotting in terms of the mean run-off. This procedure, as Mr. Foster explains, involves the assumption that the run-off for simultaneous periods at the two points is proportional to the long-term average run-off at these points. Such an assumption is not necessarily true at all times; but it may form the only feasible basis for an estimate of stream flow at a point where no records are available. In other words, the author states clearly that the method has limitations and that the limitations are more or less serious; but he does not give a scientific analysis of the limitations.

Under "Graphical Methods for Studying Stream Flow," the author indicates that duration curves may be divided into two classes. For convenience these may be called: (1) Flood duration curves; and (2) power duration curves. While Mr. Foster limits his paper to Class (2), this discussion will indicate briefly the methods by which an analysis can be made of Class (1) in the belief that a somewhat similar, fairly exact, analysis can be made of power duration curves.

NOTE.—The paper by H. Alden Foster, M. Am. Soc. C. E., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1933, by Messrs. Richard Pfaehler, and Edward H. Sargent.

[12] Lt.-Col., Corps of Engrs., U. S. Army, Fort Hayes, Columbus, Ohio.

[12a] Received by the Secretary December 1, 1933.

Due to the need for brevity, this discussion is neither full nor complete; it is not intended as a presentation of the subject, which would require a discussion of limitations. The principal limitation is that the flow must not be controlled to any great extent by lakes, dams, or otherwise, and the discussion is based primarily on humid conditions. Certain modifications are necessary in applying the method to semi-arid and arid regions.

In 1924, the author presented[13] a mathematical method to be used in connection with flood (or other) duration curves. That paper is the foundation of much of the work that is being done to-day, and it marked a turning point in the trend of thought on the subject. It did not define the limitations of the method, however, and, as a result, certain engineers, with a mathematical inclination, have applied the method to 20-yr records, and have freely predicted the 1 000-yr flood, or even the 10 000-yr flood from them. Some engineers apparently thought that this was going too far, and, consequently, have condemned the method without qualification.

There is some evidence, supported by practical considerations, which indicates that the duration curve is governed by slightly different laws for the section below 100 yr, and the section above 100 yr, which includes the rarer floods. A too free extension of curves, based on short records, above 100 yr, is questionable.

There are four semi-independent methods of obtaining the probable 100-yr flood at any given station. The Foster probability method, which requires a river record, is one of these methods. The probable 100-yr flood, Q_{100}, can be obtained for any station in the United States, with or without a record, with a fairly high degree of accuracy. Practically all such determinations can be made with a probable error considerably less than 10 per cent. If such determinations are used to check the accuracy of the Foster probability determinations based on 20-yr records, it will be found that they have a probable error of about 25 per cent.

Furthermore, about four out of five of such determinations will give values that are too low, this being due to the nature of the records, and the relative rarity of larger floods. In other words, the blind application of the Foster method to 20-yr records will give results that are erroneous and misleading. The fault does not lie in the method, but in the inadequacy of the data, which are used as a "sample."

The writer's width formula[14] for the probable 100-yr flood is:

$$Q_{100} = C\,P\,W^{1.25} \qquad (1)$$

in which, P is a rainfall index (the probable 100-yr rainfall), and W is the average width of the drainage area, in miles. It is desirable, in general, that a flood study should cover a group of stations, in the same general section of country, so that the study can be placed on a statistical basis. The width formula meets every statistical requirement, including close agreement with determinations by the Foster method when the records are more than for

[13] "Theoretical Frequency Curves and Their Application to Engineering Problems," *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[14] "The Probable 100-Year Flood," by C. R. Pettis, M. Am. Soc. C. E. (Published privately.)

40 yr. For the purpose of checking results, all river records in the section under study, and all records that are long enough to be of any statistical value, should be subjected to the Foster method, the mean formula, and the logarithmic method. When thus used and checked, the width formula is both simple and accurate.

In giving a value for Q_{100}, Equation (1) locates one point on the duration curve for the station in question. Most short-time records, such as 20-yr records, will give duration curves that are not truly representative, and that pass below Q_{100} as plotted.

The mean maximum annual flood, or the probable 1-yr flood, Q_1, may be obtained from a river record, or may be computed by the formula,

$$Q_1 = C\,P\,W^{1.05}\,L^{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, L is the length of the river, in miles. This gives the mean flood with reference to which other values on the duration curve may be plotted.

The probable 100-yr flood in terms of the mean is,

$$\frac{Q_{100}}{Q_1} = C\,W^{0.2}\,L^{-0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Now Q_1 fixes the mean line of the duration curve; Q_{100} gives an upper point on the curve; and, $\frac{Q_{100}}{Q_1}$ fixes a definite relation between the coefficient of variation and the coefficient of skew. The curve is now fixed within fairly narrow limits. It is fully fixed if a value is assumed for either the coefficient of variation or the coefficient of skew.

The mean formula for the 100-yr flood,

$$Q_{100} = C\,Q_1\,W^{0.2}\,L^{-0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

will give a probable error of less than 10% even with a 20-yr record, which generally gives a fairly good value of the mean, and it is believed a fairly good value of the coefficient of variation. The coefficient of skew is the principal element of uncertainty of the Foster method with short-time records.

The duration curve as plotted gives values that will probably be equalled or exceeded in different frequency periods. The writer prefers to use the probable maximum flood that will occur in a given frequency period, F.

The following approximate relation can be used to convert from one to the other: The probable maximum flood in a given period, F, is equal to the flood that will probably be equalled or exceeded in a period of 2.5 F yr. A consideration of this relationship suggested the logarithmic method which is indicated by the formula,

$$Q_{100} = Q_1\,(1 + 6.6\,K) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, K is obtained from the relation,

$$K = \frac{Q_{2F} - Q_F}{Q_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

An average value of K can be obtained from a river record, by using values of F of 1, 2, 4, 8, etc. This method is much simpler to apply than the Foster

method. In general, it seems to be almost, but not quite, as accurate as that method.

All formulas given in this discussion were derived rationally, one step leading to another. They have all been checked against actual river records. Collectively, they are far more accurate than any other method of flood prediction that has come to the writer's attention. Flood prediction is approaching the point where it can be classed as an exact science, the foundation being statistical.

Specifically, it is possible to construct a synthetic flood duration curve for some river point, *B*, without a record, with the knowledge that the synthetic curve will probably represent conditions better than a curve transplanted from some other point, *A*. At points down stream there is a tendency, subject to temporary reversals, for the values of $\frac{Q_{100}}{Q_1}$ to decrease decidedly.

It is believed that the laws governing power duration curves offer an interesting field of investigation, and that the analysis will be no more complicated nor difficult than was the similar analysis of flood duration curves.

W. G. HOYT,[15] M. AM. SOC. C. E. (by letter)[15a].—One phase of the duration curve, which is of national and international interest, is not discussed by Mr. Foster. In 1909 the U. S. Geological Survey prepared the first estimates of the potential power of the United States for President Theodore Roosevelt's National Conservation Commission. In making these estimates,[16] two rates of flow were used—a so-called minimum flow obtained by averaging the two consecutive 7-day periods of lowest flow in each year, and a higher rate based on the stream flow that could be assured for six months of the year. These rates are fairly comparable with those still being used by the Canadian Water Power Service. About 1919, the Geological Survey undertook a compilation of world power statistics for use by President Wilson's advisers at the Paris Conference following the World War. As detailed estimates of stream flow were not available for many countries, great refinement was impossible, and where more reliable information was not available the estimates of power were based on a flow believed to be available 75% of the time.

During the period, 1910 to 1920, there was a rapid increase in the use of the duration curve among engineers, largely, it is believed, as a result of its adoption by the New York State Water Supply Commission. On account of its many apparent advantages, the duration curve was adopted by the U. S. Geological Survey about 1920 as a basis for defining rates of flow to be used in computing water-power statistics. At that time two rates of flow—namely, the flow available 90% of the time (designated *Q*90) and the flow available 50% of the time (designated *Q*50)—were adopted as Geological Survey standards.

[15] Hydr. Engr. (Prin.), Conservation Branch, U. S. Geological Survey, Washington, D. C.

[15a] Received by the Secretary December 9, 1933.

[16] *Water Supply Paper No. 234.*

When the Federal Power Commission was created, in 1921, the Commission adopted the $Q90$ rate of flow as a basis for computing the project horsepower capacity on which administrative charges were to be levied. At a somewhat earlier date the Wisconsin Railroad Commission adopted the $Q50$ flow for estimating the horse-power capacity of sites under license by that Commission.

At the World Power Conference in London, England, in July, 1924, reports were presented on the developed and undeveloped resources of the various countries represented. These reports used no common standards for stating the power resources of rivers, and, therefore, it was impossible to make reasonably satisfactory comparisons of the resources of the different countries. After the publication of an editorial in the *Electrical World* for October 11, 1924, entitled "Wanted—A Standard Method for Rating Rivers," the American Engineering Standards Committee (now the American Standards Association), authorized the appointment of a Sectional Committee to determine bases for computing, and units for expressing, water-power resources for statistical use and for the purpose of making comparisons between different rivers, river basins, regions, and countries.

After two years of discussion, this Sectional Committee tentatively adopted the $Q90$ and $Q50$ rates of flow and recommended that these rates be used, with the gross head and an assumption of 100% efficiency, to compute the theoretical power of rivers, in kilowatts. Consideration of these rates by the American Standards Association is being held up pending international consideration.

In 1927, an International Advisory Committee on Rating of Rivers was created by the International Electrotechnical Commission. The countries making up this International Committee included Canada, Czecho-Slovakia, France, Germany, Great Britain, Italy, Norway, Sweden, Switzerland, and the United States. The United States National Committee acted as Secretariat, with the personnel of the Sectional Committee previously mentioned acting as the United States Advisory Committee. After protracted correspondence and meetings, the International Advisory Committee, at a meeting in Stockholm, Sweden, on July 4, 1930, adopted three rates of flow—namely, the flow available 95% of the time (or approximate equivalent); the flow available 50% of the time; and the mean flow. On account of difficulties in translation, these rates have not yet (1934) been officially adopted by the International Electrotechnical Commission, the French National Committee insisting thus far on defining each rate of flow as the average of the duration curves for each year rather than the duration curve for the period as a whole. It is expected that this difficulty will be straightened out soon, and that the rates recommended at the Stockholm meeting will be officially adopted by the International Electrotechnical Commission. In the meantime, the World Power Conference has adopted as a basis for its international power statistics the rates of flow as set forth, using gross head, 100% efficiency, and $Q95$, $Q50$, and the mean flows, and expressing the theoretical capacity in kilowatts. All engineers who have attempted to reconcile estimates based on rates of flow other than the duration curve will realize the importance of a national

and international standard. It is doubtful whether agreement could have been reached on the basis of any defined rates of flow other than those obtained through the use of the duration curve. Reasons justifying national and international adoption include the following: (*A*) It is in widespread use among engineers in many countries; (*B*) rates of flow based on the curve are easily defined; (*C*) defined rates of flow can be easily checked by independent computation; and (*D*) the curve presents a generalized picture of the flow and the relation of flows of various magnitude to the time covered by the record.

There is some confusion among engineers as to the relation between the duration curve and the deficiency curve. The latter may be defined as a curve the abcissa of which at any point shows the number of time units or the percentage that the flow was less than the amount shown by the ordinate of the curve at that point. A duration curve may be defined as a curve the abcissa of which at any point shows the number of time units or the percentage that the flow equalled or exceeded the amount shown by the ordinate of the curve at that point. A deficiency curve is readily convertible into a duration curve, or *vice versa,* by using the component of the time units. A duration curve may well be considered a special form of hydrograph, since it is made from exactly the same data used in the preparation of a hydrograph. It is arranged differently, however, the discharge data being plotted in order of magnitude rather than in order of occurrence. The area under a hydrograph represents the total flow regardless of whether the hourly, daily, weekly, or monthly data are plotted. Likewise, the area under a duration curve represents the total flow regardless of whether the time unit used in its preparation is the hour, day, week, or month.

The hydrograph indicates by its form the time unit that has been used. The plotted duration curve by its form does not show the time unit, and, therefore, the explanation must always indicate the time unit used. A duration curve that averages rates of flow corresponds to a hydrograph that is based on averages and is not a true duration curve any more than a hydrograph thus prepared is a true hydrograph. As the area under duration curves based on hourly or daily observations or weekly or monthly means is the same, curves based on any two of these different units will always cross each other, and the negative area will equal the positive area. For example, curves based on daily observations and monthly means will be separated at the minimum discharge end by an amount equal to the difference between the minimum daily discharge and minimum monthly mean, and in a similar manner curves will be separated at the maximum end by an amount equal to the difference between the corresponding maxima. The difference between curves based on hourly, daily, weekly, or monthly data is an indication of the magnitude of the fluctuation of the flow within the same time unit. The difference between duration curves based on daily discharge and monthly discharge, when expressed in terms of volume, will represent the amount of storage necessary to take care of variations in flow through the month over that shown by the mass curve based on monthly data.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DEFORMATION OF STEEL REINFORCEMENT DURING AND AFTER CONSTRUCTION

Discussion

BY MESSRS. F. E. RICHART, AND HOMER M. HADLEY

F. E. RICHART,[13] M. AM. SOC C. E. (by letter)[13a].—The paper by Professor Sergev performs a useful service in again demonstrating to engineers that actual stresses and design working stresses in reinforced concrete columns are rarely the same. It is hardly correct to say that a "comparison of these [Joint Committee working] stresses with the observed stresses indicates the departure of the actual from the anticipated," because no one should expect the Joint Committee formula to indicate the actual stress at just any stage of development of flow and shrinkage. On the other hand, and contrary to the author's statement, the Joint Committee formula does allow for the occurrence of stresses in columns due to a definite and reasonably large amount of shrinkage and plastic flow; also, because of a varying factor of safety, it discourages the use of small steel percentages which would result in undue steel stresses due to time yield effects. The Joint Committee formula was the logical sequel to the classic paper by McMillan in 1921,[14] in which the importance of shrinkage and flow was comprehensively treated. Tests of the effect of flow and shrinkage formed a large part of the column investigation, carried on by the American Concrete Institute from 1930 to date, and more than 170 columns of varied design have been under sustained loading and observation for 1 year to 3 years.

It would have been desirable had the author distinguished between the effects of shrinkage and flow. This could have been done by taking readings on "dummy" or unloaded specimens to determine the shrinkage. As it is, in Tables 2 and 3, the stresses are listed as due to "shrinkage and actual loads." Similarly, the modulus of elasticity of the concrete might well have been determined, instead of assuming it to be 2 000 000 lb per sq in. With

NOTE.—The paper by Sergius I. Sergev, Esq., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. William Russell Davis, and J. R. Worcester.

[13] Research Prof. of Eng. Materials, Univ. of Illinois, Urbana, Ill.

[13a] Received by the Secretary November 24, 1933.

[14] "A Study of Column Test Data," by F. R. McMillan, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst. 1921, p. 150.

the cylinder test strengths of exactly 5 000, 3 000, and 3 750 lb per sq in., as given in Table 1, the value of E should doubtless be much higher, although, as the author states, the effective value appears to be lower from the test data. This would naturally result from plastic flow.

The location of gauge lines on bars before they are placed in the form, and the reversal of the strain-gauge end for end during readings, are operations from which great accuracy can hardly be expected. Differences in manner of holding the gauge and slight bending of the bar in placing it in the form will introduce large errors. These methods would hardly be expected to support Conclusion 7, for example, which states that framing a beam into one side of a column produced flexural steel stresses of 600 lb per sq in. in compression and tension, on the respective sides of the column. This stress corresponds to four-fifths of a division of the gauge dial and is within the normal error of observation.

The break in the time-deformation curves of Figs. 2 and 4 of the paper seems hardly necessary. In Fig. 2, the points for two of the columns show no marked break, but great importance is evidently attached to the variation in readings for the third one. In Fig. 4, the break in the curve is introduced because of two observations which are about one dial division off of the regular trend of the curve. The writer can see no great significance in this part of the time-deformation curve and cannot see therein the basis for the conclusion that "concrete and steel do not act together during the first seven days after pouring, whereas the usual assumption is that they act together at all times." Certainly bond is developed at early ages. Abrams[15] has showed that the bond strength at 2, 4, and 7 days for concrete of this general grade is just as great a proportion of the compressive strength as it is at 28 or 60 days.

The statement that, in the case of columns, "the rich mix defeats the purpose for which it was intended," would seem to be a rather unfortunate generalization. While the columns of rich concrete did show steel stresses of 25 000 lb per sq in. after 2 years and 4 months as compared to 19 500 lb per sq in. for the leaner concrete, the difference is not alarming. Much greater differences in steel stresses might be expected from a variation in percentage of longitudinal steel than that due to a variation in concrete mixture.

The conclusion that "based on the allowed steel stress, the designs are unsafe," may be questioned. In the first place, the factor of safety involved in the Joint Committee formula is rather large, especially in the region of small steel percentages. Furthermore, it has been well established that initial shrinkage and yield stresses in columns have no effect upon the ultimate strength when the column is loaded to failure. The outstanding feature of such columns is the undesirable amount of shortening that may be produced by time effects under working loads.

The writer agrees with the author that the effect of shrinkage, and plastic flow as well, should be recognized and provided for in design. Obviously, a

[15] "Tests of Bond Between Concrete and Steel," by D. A. Abrams, M. Am. Soc. C. E., *Bulletin 71*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

design formula cannot contain terms involving variations in shrinkage, flow, modulus of elasticity, strength, and other properties which occur as the structure ages, but it must be devised to cover such combinations of these time and load effects as may produce unfavorable or dangerous conditions in the structure.

HOMER M. HADLEY,[16] ASSOC. M. AM. SOC. C. E. (by letter).[16a]—The results of a large amount of observational work, carried on for nearly two and one-half years, are summarized in this paper, which is one of those contributions to the store of engineering knowledge that slowly and gradually bring about a truer understanding of the behavior and ways of Nature. Professor Sergev reports reality from that greater laboratory where Nature works with full-sized models. Even if the exact conditions of testing cannot be carefully controlled, there is a compensating advantage in the elimination of errors of interpretation and judgment which may occur when reasoning from the behavior of small models to the expected behavior of larger ones. The writer would express his appreciation to Professor Sergev for his work and report of it.

The author's findings are in accord with those of the Committee on Reinforced Concrete Columns of the American Concrete Institute. Shrinkage in the concrete induces compressive stresses in the reinforcement which, depending upon the relative cross-sectional areas of the two materials, the various physical and chemical properties of the concrete, its age, etc., are of varying magnitude but, in general, are of high value when contrasted with the values commonly assumed in current design.

It is always something of a shock to find that theories one has accepted unquestioningly and has fully believed in, are not so. Some of Professor Sergev's terms and expressions indicate that he has not completely adapted himself to the change from the aforementioned Joint Committee's maximum allowable stresses to those he observed. Regardless of unconventional values, however, attention must not be diverted from the fact that the Joint Committee procedure and rules "work"— they produce structures that are staunch and safe — and, as indicated by the T-beams in the paper, may be excessively safe from the viewpoint of economy.

Undoubtedly, the reinforced concrete column must be considered in a new light, in view of these and similar observations in recent years. That in numerous cases shrinkage stresses in the concrete in combination with the column loads have stressed the reinforcement to the yield point cannot be questioned. Mere probability and the tens of thousands of concrete columns in existence assure the certainty of this; and yet the columns function successfully. If adequate lateral support and bracing are provided by the enclosing concrete, by spiral, or by numerous closely spaced binders or ties, why should not longitudinal reinforcement be stressed to the yield point when at that juncture the practically unstressed concrete is capable of assuming a great additional load?

---

[16] Regional Structural Engr., Portland Cement Assoc., Seattle, Wash.

[16a] Received by the Secretary December 2, 1933.

Table 2 reports unit stresses in the 1:1:2 columns at 850 days as 25 500 lb per sq in. in the reinforcement and 10 lb per sq in. in the concrete; and the total column load as being 160 000 lb. Should this entire load be doubled by some means and should all the increase come upon the concrete, it would result in an additional stress of 492 lb per sq in., making a total concrete stress of 502 lb per sq in., with concrete which Table 1 shows to have a cylinder strength of 5 000 lb per sq in. There does not appear to be anything alarming in this prospect, particularly when the doubled column load is purely hypothetical and has no conceivable source or origin in the building.

Toward the end of the paper Professor Sergev states that "the average column design favors the concrete, but not the steel. Based on the allowed steel stress the designs are unsafe." Furthermore, he states that when, eventually, the yield point of the steel is reached and subsequently large deformations occur with any increase in load, "the concrete will take a larger share of the imposed load, a fact which may explain why some reinforced concrete columns do not fail when seemingly they should." The suggestions in this wording are not the happiest. The evidence submitted in no wise indicates lack of safety in the columns nor incipient failure. All that is shown is a very complete lack of concordance between assumed unit stresses and actual unit stresses. It is the assumptions which are shown to be weak and in need of strengthening—not the columns.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SOME SOIL PRESSURE TESTS

Discussion

BY J. C. MEEM, M. AM. SOC. C. E.

J. C. MEEM,[3] M. AM. SOC. C. E. (by letter)[3a].—It is not an agreeable task which has been self-assigned to the writer—that of criticizing adversely this carefully and painstakingly prepared paper. There is one criticism, however, which is fundamental in its application to a soil pressure test of this scale. Due to the normal lateral pressure of soil, the results of any tests must be misleading if not valueless unless they are made on a scale sufficiently large to preclude the possibility of lateral arching, either vertical or horizontal. This applies even with greater force to kiln-dried sand which, in the writer's judgment, is the only material in which such tests should be made and final results of value computed.

The fallacy of expecting results of value from apparatus having a face of even 5 ft in width can be shown by making the following simple test or experiment. A box of 1-ft cube is constructed without top or bottom. A bottom plate, slightly larger in area than that of the box (say, 14 by 14 in.), carrying four 15-in. bolts, one near each corner, forms the loose unattached bottom. The box is filled level full with kiln-dried sand and a top plate about 10 by 10 in. in area, with bolt holes, forms the top. The bottom plate is brought to bear firmly against the bottom edges of the box by tightening the washer bearing nuts at the top. As these nuts are tightened, the sand in the box is firmly compacted; but instead of transmitting this pressure to the bottom, it is transmitted laterally, and friction is developed between the sand and the sides.

It will be seen that the development of this friction can readily be made sufficient to carry the sub-arch sand bearing on the bottom, at which time the box can be lifted as a whole and carried about without danger of collapse. If the inside faces of the box are not too smooth, the friction, of course, will be developed more readily.

It is necessarily true that if this can be done with a 1-ft cube, it can be done with a 5-ft cube; except that it is relatively more difficult to transmit by hand the pressure necessary to develop the required friction to support the 1 000-lb load on the bottom of a 5-ft cube than to support the 8-lb load

NOTE.—The paper by H. de B. Parsons, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[3] Cons. Engr., Brooklyn, N. Y.

[3a] Received by the Secretary November 29, 1933.

on the 1-ft cube. Under natural conditions this is developed by increased friction and cohesion due to moisture and to increase of depth sufficient to provide relatively larger frictional area.

If it can be shown, therefore, that lateral pressure on a 5-ft cube may be fully developed even in kiln-dried sand, the results of tests made in a box with only a 5-ft opening must necessarily be misleading and without definite value. This is true not only with kiln-dried sand, but with moist sand, since it can be readily shown that one side of a 5-ft cube of ordinary moist or

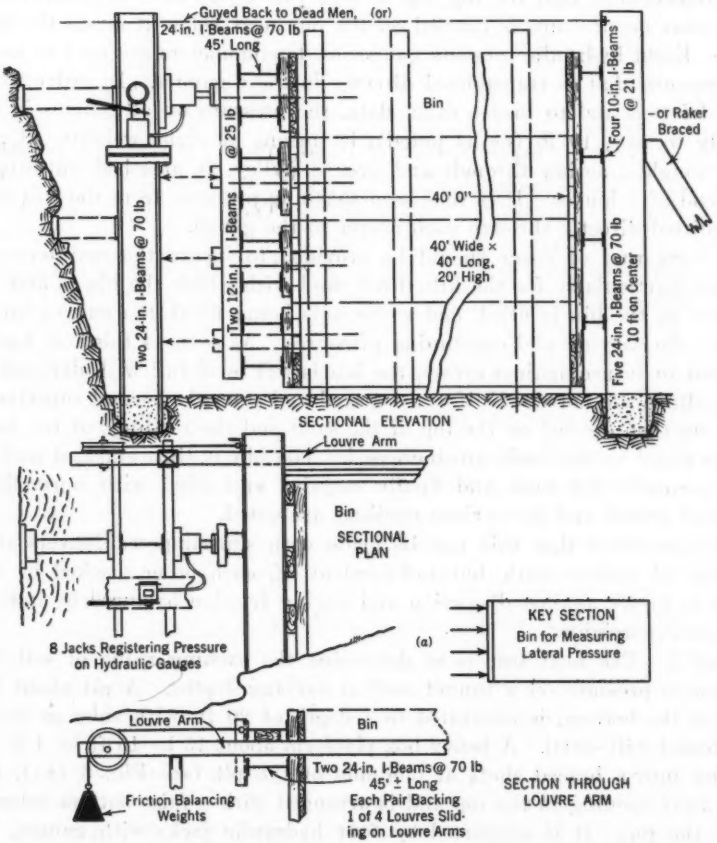
Guyed Back to Dead Men, (or)
24-in. I-Beams @ 70 lb
45' Long
Bin
Four 10-in. I-Beams @ 21 lb
or Raker Braced
@ 25 lb
I-Beams
40' 0"
Two 12-in.
Two 24-in. I-Beams @ 70 lb
40' Wide × 40' Long, 20' High
Five 24-in. I-Beams @ 70 lb 10 ft on Center
SECTIONAL ELEVATION
Louvre Arm
Bin
SECTIONAL PLAN
KEY SECTION
Bin for Measuring Lateral Pressure
(a)
8 Jacks Registering Pressure on Hydraulic Gauges
Louvre Arm
Bin
Two 24-in. I-Beams @ 70 lb 45' ± Long Each Pair Backing 1 of 4 Louvres Sliding on Louvre Arms
Friction Balancing Weights
SECTION THROUGH LOUVRE ARM

FIG. 5.—DETAILS OF SAND BIN FOR MEASURING LATERAL EARTH PRESSURES.

moulding sand may be cut vertically without developing any side pressure whatever.

For a long time the writer has advocated and proposed the making of soil tests on a scale sufficiently large to obviate the possibility of erroneous results from lateral arching or other causes. The tests or examinations, as suggested, are hereinafter noted.

Test 1.—Determination of Force and Value of Lateral Earth Pressure.—In order that this determination may be definite and convincing, it must be made

on a scale at least approximately as large as that shown, otherwise the cohesiveness of soil particles, as well as the lateral arching properties of the soil will make the tests of little or no value.

Fig. 5 shows an apparatus consisting of a bin 20 ft high, 40 ft wide, and 40 ft long, net inside dimensions, in a pit or in the open, and braced and tied across so as to leave it internally clear and free of all tie-bolts. The face of the bin is made of four louvers, each 5 by 40 ft, and backed by two 24-in., 70-lb beams to insure stability and stiffness. These beams are faced and connected so that the top one of each pair slides on a horizontal arm in order that no pressure is exerted on the one next below it or on the bottom itself. Eight hydraulic pressure gauges or dynamometers are used to measure the pressure that is transmitted directly to these gauges. In order to overcome friction and to insure exact data, the pressure on all gauges is raised equally to, say, 10 lb (or as noted), by means of eight weights. Each of these weights, acting through and over a pulley, is attached outwardly to each end of a louver. It is thus seen that any pressure from the soil will be transmitted directly through each louver to the gauge.

A light roof or cover should be provided to insure the maintenance of dryness, particularly for the kiln-dried sand with which the bin is first filled. As soon as the bin is filled, and while it is being filled, the gauges are read to note the varying and continuing pressures. As soon as this test has been repeated to insure against errors, the bin is left level full with dry sand and the readings are noted. Uniformly spaced and properly located superimposed loads are then erected on the top of the sand and the readings of the various gauges noted as the loads are increased. The bin is then emptied and filled with normally dry sand and finally emptied and filled with normally dry sand and gravel, and the various readings are noted.

It is essential that this test be made with superimposed louvers sliding on arms in contact with, but independent of, each other, backed by heavy beams to insure against distortion and having friction balanced by independently acting weights.

Test 2.—The next test is to determine the arching value of soil in its relation to pressure on a tunnel roof at varying depths. A pit about 15 by 20 ft at the bottom, is excavated to a depth of 20 ft, with sides as steep as the ground will stand. A heavy box platform about 15 by 15 ft by 4 ft high, opening into a braced shaft at one side of the pit (see Fig. 6 (*b*)), has a 10 by 10-ft opening in the top and is arranged with a false bottom below and inside the top. It is supported by four hydraulic jacks with gauges. The bin is then filled with kiln-dried sand and the increased pressure on the gauges is noted as the depth of sand is increased, or until no further increase of pressure is observed.

If or when this point or height is reached, superimposed loads are added and increases of pressure (if any) are noted. This test may be repeated with moist or normally dry sand or gravel, but it is believed that the best results will be noted with the kiln-dried sand.

Test 3.—A large pit, say, 30 by 50 ft at the top (see Fig. 6(*b*)) and perhaps 20 by 40 ft, at the bottom, and about 20 ft deep, is dug in firm non-

porous soil. A coffer-dam of the master sheet-pile type is then driven into the bottom soil with the master-piling penetrating about 5 ft and the sheet-piling just into or below the bottom. The master piles are then braced at the top with adjustable bracing.

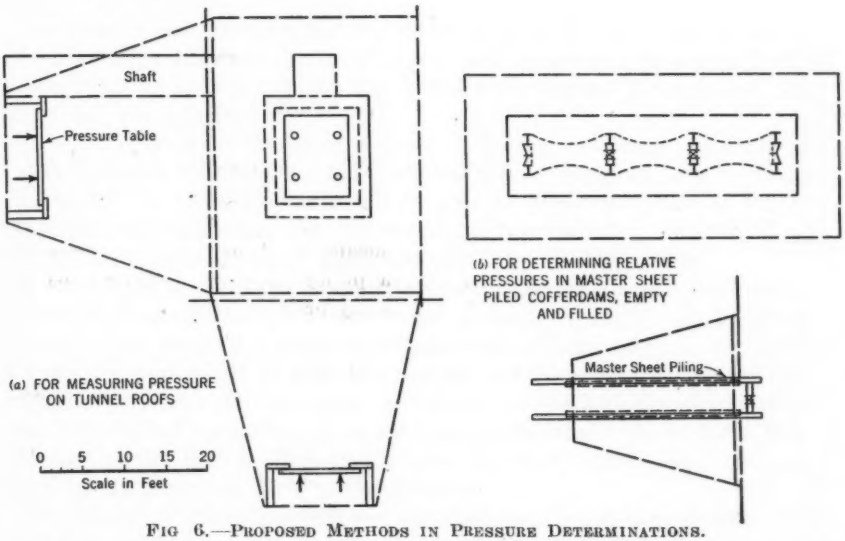
Shaft
Pressure Table
(a) FOR MEASURING PRESSURE ON TUNNEL ROOFS
0 5 10 15 20
Scale in Feet
(b) FOR DETERMINING RELATIVE PRESSURES IN MASTER SHEET PILED COFFERDAMS, EMPTY AND FILLED
Master Sheet Piling

FIG 6.—PROPOSED METHODS IN PRESSURE DETERMINATIONS.

Fill the pit outside the coffer-dam with sand, then flood it to above the level of the sand. Replace the bracing with bracing carrying pressure gauges and note the pressure. Then gradually pump the water from inside the coffer-dam and note the increase of pressure (if any) due to the lowering of the water. This test should be of incalculable value to those who contemplate building coffer-dams.

It is believed that the tremendous importance of the results from such tests would lead the author to consider the possibility of making them with co-ordination between such facilities as he may have at his command. It is in effect simply bringing his own tests out of the danger zone of a scale too small to be of value. The writer will be glad always to assist and co-operate as much as possible in making such tests.

Concerning the question of the use of the term, angle of repose, the author finds some objection because the plane of repose or rupture for normally dry or moist soils is found usually to have a curved instead of a rectilinear cross-section. The angle of repose is that angle between the normal slope of the soil after the soil above the plane of repose has slid away, and the vertical. If the line measuring this angle is curved instead of rectilinear, the angle of repose may be taken as the mean of the sum of all the angles between the tangents and the vertical.

It is certain that the only pressure possible in a retaining wall from sandy soil must come from that volume of soil sliding or tending to slide, and the measure of its cross-sectional area is that between the plane of repose and the vertical.

APPLICATIONS FOR ADMISSION AND FOR TRANSFER

The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer,* and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally, as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

The Board of Direction will not consider the applications herein contained from residents of North America until the expiration of thirty (30) days, and from non-residents of North America until the expiration of ninety (90) days from January 15, 1934.

MINIMUM REQUIREMENTS FOR ADMISSION

Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
Member	Qualified to design as well as to direct important work	35 years	12 years*	5 years of important work
Associate Member	Qualified to direct work	27 years	8 years*	1 year
Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years of important work
Fellow	Contributor to the permanent funds of the Society			

* Graduation from a school of engineering of recognized reputation is equivalent to 4 years of active practice.

† Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

FOR ADMISSION

AALTO, JOHAN AUGUST, Ardsley, N. Y. (Age 25.) Refers to E. G. Hooper, C. T. Schwarze.

ADAIR, GEORGE PARKE, Lynbrook, N. Y. (Age 26.) Field Engr., Erection Dept., McClintic Marshall Corporation, New York City. Refers to H. P. Hammond, P. G. Laurson, R. A. Pendergrass, E. J. Squire, J. H. Wagner.

BATE, STUART MILL, Los Angeles, Cal. (Age 55.) Associate Engr. with Donald M. Baker. Refers to D. M. Baker, M. H. Doyne, A. C. Everham, J. V. Hanna, F. G. Jonah, C. E. Smith, S. L. Wonson.

BROWN, WALTER EMERSON, Haddonfield, N. J. (Age 51.) With Delaware River Joint Comm. Refers to L. Bush, A. B. Cohen, F. O. Dufour, R. Farnham, G. J. Ray, H. J. Sherman.

DEMING, STEPHEN ARTHUR, Kansas City, Mo. (Age 27.) Asst. Project Engr., Missouri State Highway Comm. Refers to E. Boyce, G. W. Bradshaw, H. M. Brush, R. E. Lawrence, W. C. McNown, H. A. Rice, F. A. Russell.

DOVERSPIKE, HARRY CLYDE, St. Paul, Minn. (Age 36.) Asst. Civ. Engr., U. S. Engrs. Refers to N. W. Bowden, H. M. Hill, W. L. Kuehnle, J. S. Lambie, L. W. McIntyre, J. L. Southworth.

EASTERBROOKS, PRESTON BURT, Jr., Boston, Mass. (Age 28.) Student, Dept. of Civ. Eng., Tufts Coll., Medford, Mass. Refers to W. C. Bodycomb, H. P. Burden, G. W. Burpee, R. W. Lefavour, F. N. Weaver.

FLANIGAN, PIERCE JOHN, Jr., Baltimore, Md. (Age 22.) Refers to S. A. Becker, M. O. Fuller, H. G. Payrow, C. H. Sutherland.

GEE, ROBERT EUGENE, Los Angeles, Cal. (Age 24.) Refers to R. E. Davis, C. Derleth, Jr., B. A. Etcheverry, R. M. Fox, S. T. Harding, C. G. Hyde, G. E. Troxell.

GOODHEART, EDMUND JAMES, Los Angeles, Cal. (Age 29.) Refers to J. H. Brillhart, T. S. Byrne, L. N. Fisher, H. B. Friedman, T. S. Needels.

HATCH, LORANUS PENDLETON, Cambridge, Mass. (Age 28.) Research Fellow in San. Eng., Harvard Eng. School. Refers to G. M. Fair, A. Haertlein, E. H. Sprague.

HATHAWAY, GAIL ABNER, Kansas City, Mo. (Age 38.) Hydr. Engr., U. S. Engr. Dept. Refers to G. H. Canfield, J. W. Cunningham, G. C. Haydon, D. C. Henny, H. S. Rogers, J. C. Stevens, C. W. Sturtevant.

HELLSTROM, BO MANNE, London, S. W. 1, England. (Age 43.) Partner, Vattenbyggnadsbyran (V. B. B.), Cons. Civ. Engrs., Stockholm, Sweden. Refers to W. J. E. Binnie, H. E. Gruner, H. G. Howard, F. Palmer, G. B. E. Truscott, F. Vogt, P. W. Werner.

HOFMANN, OLIVER DIMMITT, Eagle Rock, Cal. (Age 22.) Refers to R. R. Martel, F. Thomas.

HOLLAND, ELWOOD WILLIAM, Banning, Cal. (Age 26.) Jun. Engr., Metropolitan Water Dist. of Southern California. Refers to J. B. Bond, B. A. Eddy, R. M. Fox, F. W. Hough, D. M. Wilson.

HUBBARD, JAMES WILLIAM, Smethport, Pa., (Age 35.) Supt. and Gen. Mgr., Smethport Water Co. Refers to J. L. Herber, D. Moomaw, W. S. Moore, H. A. Olson, J. R. Pollock, H. G. Wray.

KELLY, WILL LEE, Ft. Worth, Tex. (Age 40.) With County Engr.'s Office. Refers to W. H. Bosier, J. H. Brillhart, F. D. Hughes, J. H. Knox, F. E. Lovett, W. M. Powell.

KLINGENBERG, ALBERT CHARLES, Baltimore, Md. (Age 29.) Catalano Constr. Co. Refers to H. H. Allen, J. E. Greiner, J. K. Knoerle, H. G. Perring, O. H. Schroedl.

KRZYWIEC, VLADIMIR JOSEPH, New York City. (Age 49.) Refers to B. A. Bakhmeteff, V. T. Givotovsky, N. B. Jacobs, C. D. Kaminsky, A. V. Karpov, C. P. Melioransky, R. Modjeski, L. S. Moisseiff.

LaMOTTE, FRANK WILLIAM, Columbia, S. C. (Age 30.) Engr. Refers to S. C. Dreyfus, A. Epstein, R. C. Johnson, G. P. Jones, T. K. Legare, W. E. Rowe.

MacLEOD, COLIN HUBERT, White Plains, N. Y. (Age 31.) Asst. Engr. to W. W. Young, Cons. Engr. for Westchester County San. Comm. Refers to E. H. Feldmann, F. J. Laverty, C. J. Suhr, J. W. Van Denburg, A. M. Wyman.

McINTOSH, WALTER TOWNSEND, Tenafly, N. J. (Age 47.) Private practice (developing estates), New York City. Refers to W. B. Hunter, H. T. Immerman, J. Meltzer, T. Merriman, E. A. Prentis, J. F. Sanborn, L. White.

McMANAMNA, THEODORE LOUIS, Toronto, Ont., Canada. (Age 30.) Field Engr. and Mgr., International Water Supply, Ltd. Refers to E. Boyce, R. S. Charles, J. G. Gordon, Jr., J. P. Lawlor, R. E. Lawrence, F. A. Russell, R. R. Schweitzer.

McMULLEN, ONIE EL ROY, Lansdowne, Pa. (Age 43.) Regional Structural Engr., Portland Cement Association, Philadelphia, Pa. Refers to A. J. Boase, W. Easby, Jr., J. W. Follin, C. T. Johnston, M. den H. Kolyn, H. E. Riggs, E. C. Shoecraft.

MEHLA, RAMESH SUMANT, Ahmedabad, India. (Age 27.) San. Engr., Duncan Stretta & Co., Bombay, India. Refers to T. R. Camp, H. N. Ogden, C. M. Spofford, P. H. Underwood, L. C. Urquhart.

MURRAY, MATTHEW SWEETMAN, Kansas City, Mo. (Age 50.) Director of Public Works. Refers to E. B. Black, C. W. Brown, C. E. Johnston, T. J. Strickler, N. T. Veatch, Jr., R. W. Waddell.

NARVER, DAVID LEE, Los Angeles, Cal. (Age 43.) Holmes & Narver, Inc. Refers to D. M. Baker, R. W. Binder, O. G. Bowen, R. V. Labarre, A. F. J. Miller, R. P. Miller, H. Z. Osborne.

RALSTON, HARRY JAMES, Jersey City, N. J. (Age 25.) Eng. Asst. with Alexander Potter, New York City. Refers to T. A. Appleton, F. A. Barbour, F. W. Haley, A. Potter, S. A. Thoresen, R. S. Watts.

ROVER, HENRY JOHN, Brooklyn, N. Y. (Age 41.) Asst. Engr., Sewer Sec., Div. of Design, Bureau of Eng., Dept. of Public Works, Borough of Manhattan, N. Y. Refers to H. R. Codwise, A. Dick, R. A. MacGregor, C. M. Pinckney, J. C. Riedel, A. W. Tidd.

TRUDEAU, ARTHUR GILBERT, Leonia, N. J. (Age 31.) Lieut., Corps of Engrs., U. S. Army. Refers to C. Derleth, Jr., C. G. Hyde, C. W. Kutz, E. H. Marks, O. Singstad, W. C. Weeks.

WENTWORTH, CHESTER KEELER, St. Louis, Mo. (Age 42.) Associate Prof. of Geology, Washington Univ., St. Louis; also Geologist, U. S. Engr, Office, Kansas City, Mo., Refers to G. B. Archibald, W. Gerig, W. W. Horner, F. E. Matthes, E. O. Sweetser.

WISEMAN, EUGENE ROLAND, North Troy, N. Y. (Age 39.) Asst. Prof., Dept. of Civ. Eng., Rensselaer Polytechnic Inst. Refers to E. R. Cary, L. W. Clark, H. B. Compton, T. R. Lawson, W. W. Rousseau, H. O. Sharp.

FOR TRANSFER

FROM THE GRADE OF ASSOCIATE MEMBER

BARNETT, JOSEPH, Assoc. M., Mt. Vernon, N. Y. (Elected March 15, 1926) (Age 37.) Chf. Draftsman, Eng. Design Div., Westchester County Park Comm., White Plains, N. Y. Refers to E. Anderberg, F. E. Foss, C. A. Garfield, A. G. Hayden, R. M. Hodges, L. G. Holleran, F. F. Skinner.

COWIE, GEORGE DURNO, Assoc. M., South Orange, N. J. (Elected Jan. 15, 1923.) (Age 45.) Officer-in-Charge, New York Field Station, U. S. Coast and Geodetic Survey. Refers to W. Bowie, C. L. Garner, J. H. Hawley, G. C. Mattison, R. S. Patton, H. A. Seran, P. C. Whitney.

KEAST, SCHUYLER SHELDON ALBERT, Assoc. M., Philadelphia, Pa. (Elected Nov. 21, 1921.) (Age 43.) Asst. Engr., Dept. of City Transit. Refers to B. Franklin, M. Golder, S. Harris, H. S. Hipwell, L. B. Manley, C. W. Palmer, C. H. Stevens.

LORENCE, WALTER ERNEST, Assoc. M., Metuchen, N. J. (Elected Aug. 18, 1930.) (Age 40.) Capt., 29th Engrs., U. S. Army. Refers to G. D. Cowie, H. S. Gladfelter, C. F. Maynard, W. A. K. Parkin, P. S. Reinecke, B. A. Ross, W. D. Waltman, W. C. Weeks.

PITTARELLI, EMILIO, Assoc. M., New York City. (Elected June 9, 1930.) (Age 35.) Gen. Mgr., Sewer Construction (relief work), City of New York. Refers to W. Heyman, J. Meltzer, G. Paaswell, A. I. Raisman, R. Ridgway, J. F. Sanborn, D. C. Serber, D. C. Waite.

FROM THE GRADE OF JUNIOR

DARLING, JAMES ARTHUR, Jun., Newark, N. J. (Elected March 14, 1927.) (Age 32.) Draftsman, The Port of New York Authority, New York City. Refers to A. P. Andersen, L. E. Andrews, W. A. Cuenot, H. W. Heilmann, C. M. Noble.

HANRAHAN, FRANCIS JOHN, Jun., State College, Pa. (Elected Feb. 10, 1930.) (Age 32.) Instructor, Pennsylvania State Coll. Refers to P. B. Breneman, H. K. Kistler, R. O'Donnell, R. L. Sackett, H. B. Shattuck.

HUTCHINSON, ROBERT PADDOCK, Jun., Seattle, Wash. (Elected March 14, 1927.) (Age 32.) Structural Designer, The Austin Co. Refers to W. R. Engstrom, L. P. Hover, R. A. Huestis, C. C. May, C. C. More, M. O. Sylliaasen.

JOHNSON, IRVING LAURENCE, Jun., San Francisco, Cal. (Elected June 27, 1932.) (Age 27.) Assist. Bridge Constr. Engr., Triangulation Div., San Francisco-Oakland Bay Bridge. Refers to D. Butler, D. R. Cate, B. C. Gerwick, J. W. Gross, A. G. Proctor, C. F. Swigert, D. R. Warren, J. B. Wells.

MEGGITT, SAMUEL EDWARD, Jun., London, England. (Elected May 19, 1924.) (Age 32.) Senior Asst. with B. L. Hurst, Chartered Civ. Engr. Refers to A. E. Brook, M. E. Habershon, J. Husband. (Applies in accordance with Sec. 1, Art. 1, of the By-Laws.)

OSBORN, LEWIS KNOWLES, Jun., Long Beach, Cal. (Elected Oct. 14, 1929.) (Age 29.) Designer, T. C. Kistner & Co., Los Angeles, Cal. Refers to H. G. Balcom, J. L. Edwards, L. R. Hjorth, C. C. More, W. F. Way, W. T. Wright.

PESCHEL, HARRY WILLIAM, Jun., Philadelphia, Pa. (Elected June 10, 1929.) (Age 28.) Draftsman, Bureau of Eng. Refes to H. M. Beaumont, O. H. Gentner, Jr., W. McCormick, W. B. Nissly, W. K. Wyatt.

ROBERTS, PAUL OSBORNE, Jun., Brookhaven, Miss. (Elected Feb. 24, 1931.) (Age 31.) Bridge Inspector, Mississippi Central R. R. Refers to R. L. Ehrlich, J. S. Hazelton, F. I. Louckes, H. V. Pittman, C. S. Sample, L. A. Weeks, S. L. Wonson.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.